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ISSUE No. 1

PRACTICAL MATHEMATICS

THEORY AND PRACTICE WITH MILITARY AND INDUSTRIAL APPLICATIONS

BASIC ARITHMETIC

The Fundamental Operations

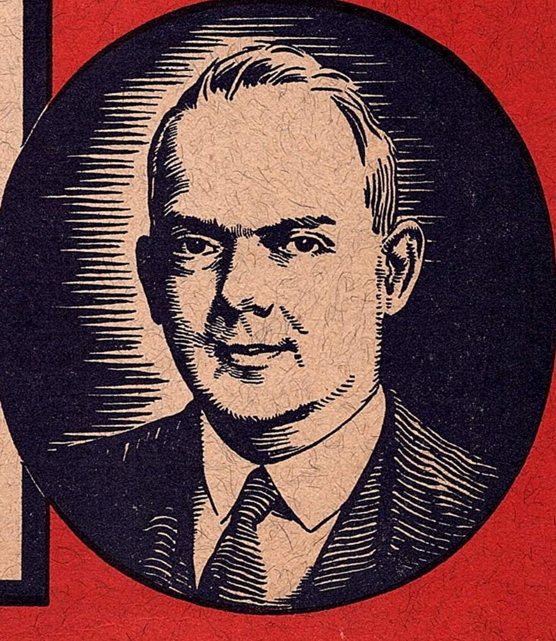
Addition • Multiplication
Subtraction • Division

Common Fractions
Systems of Weights & Measures
Numbers through the Ages

—ALSO—

Mathematical Tables and Formulas
Glossary of Mathematical Terms
Self-Tests and Arithmetic Problems

REGINALD STEVENS KIMBALL



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EDITOR: REGINALD STEVENS KIMBALL ED.D.

Issue No. 1 PRACTICAL MATHEMATICS Volume No. 1

ISSUE 1 **Practical Mathematics** VOLUME 1

REGINALD STEVENS KIMBALL, *Editor*

TABLE OF CONTENTS

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PRACTICAL MATHEMATICS

Will Feature Step-by-Step Instructions by Leading Experts in All 14 Major Branches of Mathematics with Practical Applications to War and Industry

The Complete Course Will Consist of 14 Lecture-Groups or Issues. Below Is a Partial List of the Subjects that Will Be Covered.

ADDITION

Simple Rules and Short-Cuts to Addition.
Checking Accuracy in Addition.
The Secret of Mental Addition.
How to Determine the Distance Around a Lake.
How to Determine by Easy Calculation the Number of Board Feet Required for the Construction of a Floor.
Simplified Methods for Determining the Total Resistance of an Electric Circuit.

SUBTRACTION

The Shop Method of Subtraction.
Checking Correctness in Subtraction.
How to Use Subtraction in Computing Production Figures.
An Easy Way to Find the Difference Between the Boiling Point and Melting Point of Certain Essential Metals.
Rapid Calculations for Determining the Difference in Speeds of Two Machines.

MULTIPLICATION

How to Use Multiplication as a Rapid Form of Addition.
Short Cuts in Multiplication.
Checking Accuracy in Multiplication.
A Quick Way to Find the Circumference of a Circle.
Using Multiplication in Making up a Pay-Roll.
Rapid Calculations of the Speed with which Given Articles may be Produced.
How to Find the Quantities of Raw Materials Needed to Produce Certain Alloys.
Practical Military Problems that may be Solved by Multiplication.

DIVISION

How to Divide.
How to Perform Long Division.
The Magic of Trial Divisors.
Determining the Divisibility of Numbers.

(Continued on next 3 pages)

Your Instructors Chosen from Famous Universities

Most Distinguished Faculty of Mathematicians Ever Assembled.

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Checking Correctness in Division.

How to Find the Number of Gallons of Oil Needed to Fill a Tank.

Easy Calculations for Dividing Labor Among a Number of Workers.

How to Determine the Length of Struts of a Military Bridge.

How to Figure the Pitch of Screw Threads.

How to Calculate the Number of Ribs Required for an Airplane Wing of Given Dimension.

How to Find the r.p.m. of a Drive Shaft with Different Sets of Gears Engaged.

COMMON FRACTIONS

What Fractions Are and How to Use Them.

Cancelling as a Short-Cut in Using Fractions.

The Addition and Subtraction of Fractions.

The Lowest Common Denominator and Easy Ways to Use It.

Multiplication and Division of Fractional Expressions.

Proper and Improper Fractions.

An Easy Way to Determine the Unthreaded Part of a Pipe Nipple.

Rapid Calculations for Determining the Distance Between the Centers of Holes in a Template or Terminal Strip.

How to Determine the Pitch of Rivets in a Lap Joint.

How to Find Overall Length of an Irregular Strip of Metal.

How to Determine the r.p.m. of a Driven Pulley.

Fourier's Theorem of Harmonic Analysis.

How to Determine the Crest and Depth of American Standard Screw Threads.

DECIMAL FRACTIONS

How to Convert Common Fractions into Decimal Fractions.

Simplified Addition and Subtraction of Decimals.

Contracted Multiplication and Division of Common Decimals.

How to Determine the Number of Places to which Decimal Computation Should be Carried.

The Table of Decimal Equivalents and How to Use It.

How to Determine the Overall Dimensions of Figures Shown on Blueprints.

How to Determine the Length of Struts in a Steel Structure.

Rapid Calculations for Determining the Distance a Plane Can Fly on a Given Amount of Fuel.

How to Measure Pitch by Use of the Micrometer.

How to Compare the Addendum and Dedendum of a Spur Gear.

Practical Problems in Electrical Engineering.

AVERAGES

Short-Cuts in Finding Averages.

Easy Ways to Compute the Mechanical Advantages of a Machine.

How to Determine the Average Rate of Expansion of a Casting While Heating.

How to Determine the Normal Output of a War Production Plant.

Simple Ways to Find the Average Resistance of Various Types of Electric Wire.

Computing the Number of Raw Unfinished Parts Required by a Shop-Worker in the Course of a Day.

LOGARITHMS

Understanding Logarithms, What They Are, and How to Use Them.

How to Express Powers by the Use of Exponents.

The Properties of the Positive and Negative Powers of 10.

How to Use the Log Tables in Performing Short-Cut Operations in Engineering and Military Problems.

How to Find Reciprocals by Using Logarithms.

How to Solve Proportion Problems by Using Logarithms.

RATIO AND PROPORTION

What We Mean by Ratio.

What We Mean by Proportion.

How to Solve Problems Involving Axle and Gear Ratios.

How to Determine the Height of Radio Towers, Mountains, and Other Unattainable Objects.

The Use of Ratio and Proportion in Solving Navigation Problems.

How to Determine the Developed Horsepower of an Engine.

Solving Military Problems by the Use of Ratio and Proportion.

THE SLIDE RULE

How the Slide Rule Makes Difficult Mathematics Operations Simple.

Multiplication of Numbers with the Slide Rule.

How to Use the Slide Rule in Dividing Numbers.

Rapid Methods of Determining the Square Root and Cube Root of a Number with the Slide Rule.

Using the Slide Rule to Find Reciprocals.

How to Use the Slide Rule in the Machine Shop.

The Important Uses of the Slide Rule as a Military Tool.

Solving Construction Engineering Problems with the Slide Rule.

Easy Methods of Determining the Cost per Kilowatt-Hour of Electric Current by Use of the Slide Rule.

BASIC ALGEBRA

Why Many Industrial and Technical Jobs Can be Done More Intelligently with a Knowledge of Algebra.

Laws of the Fundamental Operations in Algebra.

Algebra as a Short-Cut to Mathematics.

How to Find the Factors of Algebraic Expressions.

A Practical Electrical Problem: How to Calculate the Total Resistance of a Network of Conductors.

A Simple Way to Determine the Velocity of an Airplane in Miles per Hour.

How to Find the Length of Belt Needed for Two Pulleys.

How to Solve Gear Formulas.

How to Determine the Proportions of Sand, Stone, and Cement Needed to Build a Concrete Wall of Given Dimensions.

Ohm's Law.

Using Powers and Roots to Determine the Cross Section of a Wire in Circular Mils.

How to Find the Roots of a Number.

How Logarithms Are Derived.

ADVANCED ALGEBRA

How to Find Unknown Quantities by Adding, Subtracting, or Substituting in Simultaneous Equations.

Solution of Practical Problems by the Use of Simultaneous Equations.

Solving Equations by the Use of Determinants.

A Rapid Way to Determine the Diameter of Wire Required to Carry a 3 Ton Load.

How to Determine the Drag of an Airplane Wing.

Methods for the Solution of Quadratic Equations.
Various Forms of Roots of a Quadratic Equation.
The Discriminant and How We Use It.
Application of Equations to the Solution of Wheatstone's Bridge Problem.
How to Determine the Area of a Site for an Army Camp.
Simple Development of Formulas for Expressing Permutations and Combinations.
How to Determine the Number of Committees that may be Selected from a Given Body of Men, with the Number of Members on Each Committee Specified.
How to Use a Formula to Expand a Bionomial.
Applications of the Binomial Theorem to Important Electrical Problems.

PLANE GEOMETRY

How to Construct Angles, Triangles, Parallelograms, Trapezoids, Pentagons, Hexagons, Circles, and Other Geometric Figures.
How to Measure Angles by a Simple Process.
How to Measure the Area of a Triangle, a Square, and Other Figures.
What Is a Circle?
An Accurate Explanation of the Parts of a Circle.
Why the Hyperbola, the Parabola, and the Ellipse Are Called Conic Sections.
How to Determine Whether the Foundation for a Building Is Accurately Laid.
An Easy Way to Compute the Length of a Metal Tank.
Simple Methods for Determining the Amount of Wire Needed to Support a Pole of Given Height.
A Rapid Method for Determining the Length of a Lake.
How to Find the Diameter of a Pulley.
How to Compute the Cost of Silvering a Circular Mirror.
How to Find the Ground Speed, Angle of Drift, and Gliding Angle of a Bomber. (Vectors)
How to Find the Cross-Sectional Area of a Steel Column.
How to Bisect a Line.
How to Divide a Line into Equal Parts.
How to Locate the Center of a Circle.
How to Determine the Mean Proportional.
The Pythagorean Theorem and Its Significance in Mathematics.

SOLID GEOMETRY

What is Meant by Intersecting Planes.
How to Locate Points in Space.
How to Figure the Area of Each Face of a Pyramid.
How to Determine the Volume of a Sphere.
The Use of Coördinate Points in Mechanical Drawing.
How to Interpret Blueprints.
How to Determine the Number of Square Inches that Must be Removed from an Iron Bar to Cut It Down to an Octagon.
Important Applications of Solid Geometry to Construction and Building Problems.
Practical Applications of Solid Geometry to Navigation and Avigation.

TRIGONOMETRY

The Trigonometric Functions — What We Mean by Sine, Tangent, and Secant.
How We Find the Trigonometric Functions.
Trigonometric Identities and How They Are Used.
How to Determine the Angle of Pitch for a Roof.
How to Find the Angle for a Taper.
How to Calculate the Distance Across the Internal Space of a Dovetail.
How to Measure Angles by Degrees, Minutes, and Seconds.
An Easy Method for Converting Degrees, Minutes, and Seconds into Radians.
The Use of Theodolites and Sextants, and the Vernier Scale.
The Accurate Measurement of Inaccessible Objects.
How to Use the Military "Service" Protractor.
An Easy Way to Determine the Distance Across a Body of Water.

CALCULUS

Introduction to Calculus

Calculus is the Most Direct Way of Solving Many Problems.
Calculus Provides a Means of Solving Problems that Can Be Solved in No Other Way.
Constants, Variables, and Functions, Their Meanings, and Their Relationships.

Differential Calculus

The Role Played by Trigonometry in the Study of Calculus.
A Study of the Curve and the Derivative.
Easy Rules for Finding the Derivative.
Graphic Illustrations of Derivatives.
Application to Area Problems.
The Meaning and Use of Higher Derivatives.
Application of Calculus to Electrical Problems Involving Resistance, Voltage, and Transmission of Current.
Application to Gunnery:
- The Velocity of a Bullet.
- The Time It Takes for a Bullet to Reach a Target.

Application of Calculus to Important Industrial and Manufacturing Problems.

Integral Calculus

How Integral Calculus Differs from Differential Calculus.
Easy Rules for Integrating by the Use of Fundamental Formulas.
The Usefulness of Integration in Solving Transportation and Military Problems.
How to Determine the Center of Gravity of an Irregular Figure by the Use of Calculus.
How to Determine the Moment of Inertia by Simple Calculus.
How Calculus May Be Used to Determine the Amount of Pressure Upon a Given Body of Water.
Pascal's Law.
How to Compute Work Problems.
Hooke's Law.

MEASUREMENT

Measuring Instruments of Lengths and Angles

Non-Precision Instruments:
- Tape
- Steel Rule
- Calipers (Inside and Outside)
- Protractor
- Square

Precision Instruments:
- Vernier Scales and Calipers
- Micrometers
- Dial Gage
- Vernier Protractor
- Transit
- Hoke and Johansen Gage Blocks

Optical Methods:
- Measuring Microscope
- Optical Flats

Measuring Instruments of Areas and Volumes

Areas:
- Planimeter
- Graphical Method by Use of Cross-Section Paper.

Volumes — Wet:
- Graduated Glassware
- Burettes
- Cylinders

(*Continued on next page*)

Graduates
Volumetric Flasks
Pipettes

Volumes — Dry:
Tables of Dry Measure.

Measuring Instruments for Forces

Utility Balances:
Roberval Balance
Torsion Balance
Fairbanks Scales

Precision Balances:
Analytical Balance

Units of Force and Mass:
English Engineering Units
Metric Units (C.G.S., M.K.S.)
Torque
Definition
Units

Specific Gravity and Density:
Hydrometers
Westphal Balance

Pressure:
Units

Measuring Instruments:
Atmospheric Pressure—Mercury Barometer
Aneroid Barometer

Pressures Relative to Atmospheric:
Dead Weight Tester as a Primary Standard
Manometers
Bourdon Tube Gage
Engine Indicator
Air Density
Humidity (Absolute and Relative)
Air-Fuel Ratio.

Measuring Instruments of Stress and Strain in Materials

Testing Machines:
Static and Impact Tests for Tension, Compression, Shear-Torsion
Hardness Testing
Brinell
Rockwell
Hooke's Law and the Elastic Limit
Young's Modulus of Elasticity
Poisson's Ratio
Tables for Common Metals

Work, Energy, and Power

Measurement of Work Done:
Methods
Units
Mechanical Advantage

Measurement of Power:
Prony Brake
Wood Blocks and Lever Arm
Rope Brake

Electric Generator and Load
Sprague Dynamometer
Airplane Engine Testing
Units

Special Measuring Instruments

Altitude Meter
Pilot-Tube Air-Speed Indicator
Tachometer
Velocities and Accelerations
Exposure-Meter
Illuminator

Precision of Instruments

Sources of Error
Significant Figures
Absolute Error
Percentage or Relative Error

Differential Equations

APPLIED MATHEMATICS

MACHINE-SHOP PRACTICE AND CONSTRUCTION ENGINEERING

Machine Shop

How to Compute Mounting of Parts on Adjustable Angle Plates
Milling Teeth
Screw Threads
Gears
Planetary Grading
Plain and Differential Indexing

Construction Engineering

Steps Required to Produce a Finished Structure
Behavior of Materials Under Load
Treatment of Bodies in Equilibrium and Accelerating
Stresses and Strains Caused by Temperature Changes
Impact Loading, Impulse and Momentum, Energy
Structural Members Designed to Resist Bending or Twisting
Columns
Riveted, Welded and Wood Connections

HEAT AND CHEMISTRY

Heat

Definitions   Measurement
Expansion with Heat of Solids, Liquids, and Gases
Latent Heat   Specific Heat
Calorimetry
Computation of Temperature
Heating of Buildings
Mechanical Equivalent of Heat

Chemistry

Chemical Units of Weight
Chemical Formulas and Equations
Analysis and Synthesis
Proportions
Molecular Formulas of Simple and Compound Substances
Thermo-Chemical Equations
Solubility
Vapor Density and Pressures
Standard, Semi-Normal Solutions

ELECTRICITY

Units of Measurement

Coulomb   Calorie
Ampere   Thermal Capacity
Volt   Henry
Ohm   Farad
Watt   Frequency
Joule   Multi-Unit System
Kilowatt Hour

Ohm's Law

Resistance in Series and in Parallel
Solution of Series Circuits and Parallel Circuits
Power

Kirchoff's Laws

Statement of the Laws
Steps in Application
Three-Wire Distribution System

Condensers

Calculation of Capacity
Condensers in Series and in Parallel
Capacitative Reactance

Alternating Current

Resistance
Inductive Reactance
Impedance
Power Factor
Time Constant
Effective, Peak, and Average Values

Applications to Radio

Resonant Circuits
Tubes in Series and Parallel
Coil Design

NAVIGATION, AVIGATION, AND GUNNERY

Navigation

Plotting a Course
Plotting Latitude and Longitude
Astronomical Navigation
Sound Ranging

Direction Finding

Map Reading

Gunnery and Ballistics

Determination of Direction
Elements of Exterior Ballistics
Errors and Dispersion

GENERAL APPLICATIONS AND COMPREHENSIVE INDEX

Problems in Application

Constituting a Complete Review
Index to all Subjects Included.

IN EACH ISSUE

History of Mathematics
Odd Problems for Off Hours
Strange Ways with Numbers
Review Tests and Exercises

AOH

INTRODUCING THE EDITOR

SOME years ago the teacher of a Newport, R. I., high school was confronted by the strange situation of his prize student in mathematics refusing to continue his studies in that subject. The young man had brilliantly completed three years of requirements and was the obvious candidate for the mathematics medal, awardable at the end of four years. The pupil, when pressed for an explanation, admitted to an inherent distaste for the subject. A few years later, Raymond Clare Archibald, then president of the American Mathematical Society, came across a similiar problem at Brown University. He recognized the unusual aptitude of one of the members of his class and suggested advanced courses as a preparation for a mathematics teaching career. Archibald's recommendations were politely but firmly rejected. His protege outlined plans for an exhaustive study of English literature and confessed to an ambition to write the great American novel.

This young tyro, who was blasting professorial hopes in the early '20's, was Reginald Stevens Kimball. His youthful aspirations and his record in education provide a story which is applicable to America's war training problem. As Kimball's dislike for mathematics served for a time to subordinate his natural abilities, so may many a young man's phobia for figures black out his logical contribution to the war effort.

Kimball was born in 1899, at Newport, and received his elementary and secondary education in the schools of that city. At Brown University, he earned his Bachelor of Arts in 1921, and a Master of Arts in 1922, and was elected to membership in Phi Beta Kappa, national honorary scholastic fraternity. It is worth while noting here that, while at Brown, he was one of fifteen students selected to be given special training in mathematics in conjunction with the World War I training program for officers. The war ended before this schedule could be put into operation. While studying for his Master's degree, Kimball turned to teaching, accepting his first appointment at the Hope Street High School in Providence and, later, assisting for a short period at the School of Education at Brown. When the school year ended in 1922, he was assigned a role by the Massachusetts State Department of Education in the task of converting its normal schools into teachers' colleges. In conjunction with this, he was located at the Worcester State Teachers College as assistant professor of Mathematics and History. In 1923, he was made a full professor and head of both departments.

In 1925, the American Education Press placed Kimball in charge of their research department, where he made a national survey of school curriculum. During his stay with AEP, he edited the *My Progress Books* series and the *Current Events Year Book*. The year 1928 found him at Harvard where he later obtained his Master of Education degree. From 1930 to 1939, Kimball devoted himself to the Massachusetts school system. He was superintendent of schools for the Brookfield

district until 1936, and later assumed a similar post at Monson. While at Monson, Kimball lectured for the Division of University Extension of the State of Massachusetts and then, in 1940, joined the staff of the School of Education of New York University. From the same institution, he received his Doctor of Education degree in 1941, and election to membership in Phi Delta Kappa and Kappa Delta Pi. In the fall of 1942, he came to the National Educational Alliance to organize and edit the PRACTICAL MATHEMATICS magazine.

Kimball's twenty years in the educational field have been replete with achievements which justified the prophecies of his early tutors. He is the creator of the Kimball Solid Geometry Tests, which received nation-wide acclaim. He recently developed a slide rule which can be used for addition and subtraction as well as the conventional computations. In a period when a knowledge of mathematics is a military necessity, he is able to offer a practical approach built upon a sound academic background.

FRANK W. PRICE
General Editor
National Educational Alliance

CHATS WITH THE EDITOR

MANY of you who have subscribed for PRACTICAL MATHEMATICS have been writing to me of the reasons for your interest in the subject. Some, who haven't paid any attention to mathematics since their schooldays, are now faced with the necessity of knowing how to use mathematics in their daily tasks. Others have had occasion to use some of the simpler forms of mathematics, but now find that war-time demands upon them make it essential that they extend their knowledge.

In preparing this series on PRACTICAL MATHEMATICS, my colleagues and I have had both groups in mind. We are endeavoring, in these 14 issues, to select from the whole range of mathematical knowledge just those items which can be of immediate and practical benefit to you in your war-time occupation.

You may be surprised that we have begun quite so "far back". I can imagine that some of you, on noting the title-words on the cover of this first issue, have exclaimed " 'Basic Arithmetic'? Why, I had all that stuff back in the grades!" Yes, of course you did, but, if you are at all like the average man or woman, you've grown "rusty" on some parts of the subject.

No house is any stronger than its foundations. If the underpinnings are weak, the whole structure is shaky. My own experience in meeting face to face in the classroom men and women who have been trying to "refresh" themselves on the subject of mathematics has shown me that the person who thinks he "just can't do mathematics" is usually laboring under that impression because he is weak in the fundamentals upon which all else depends.

At a recent meeting of mathematicians held at Columbia University, one prominent professor of education declared, "Much of our failure to understand some of the problems we have to meet in every-day life is due primarily to a lack of knowledge and appreciation of the simplest kinds of arithmetic." Another speaker cited

the lack of arithmetical skills on the part of naval aviation cadets whom he had been teaching.

The experience of the United States Army and Navy in recruiting men for the various armed services has also brought this fact to light. Experts in the various training schools are pointing to the fact that their task in helping these men prepare for specialized jobs is greatly handicapped because the men haven't a sufficient knowledge of simple arithmetic.

Industry, as well as the military and naval forces, is calling for "specialized applications of mathematics, with stress on basic problems and skills". These skills are listed as: "accuracy and speed in the fundamental operations of arithmetic; a knowledge of geometric forms, of direct and indirect measurements, of scale drawings and graphic representations; the ability to analyze problems, and to apply mathematical principles in many concrete situations".

Through the pages of PRACTICAL MATHEMATICS, we shall offer you an opportunity to acquire all of these. First, though, we urge you to look to the foundations. Much of the material in this first issue is not new to you, but there is undoubtedly a great deal which you have forgotten or which you never really understood. It is presented here in simple, easily remembered form, with some sidelights that are not usually taught in the classroom.

If you feel that you are "pretty good" at arithmetic, start off with the test on page 51. Chances are that you'll find a few things there that need brushing up a bit, or that you'll find it will take you longer to complete the test than the time limit suggested. If you find you're "stuck" on any part of it, the small print below that group of exercises will direct you to the part of the issue that will help you to gain a better understanding.

If you're slow, practice examples of the same type, timing yourself until you have developed the speed which you need. In the air or on the job, an airplane pilot or a lathe mechanic isn't of much use if it takes him "all day" to figure something out. It's the man with a fair degree of speed and a high degree of accuracy who's going to help win this war.

That's why we're devoting this first issue to the simplest facts. In succeeding issues, we'll guide you through more advanced operations in arithmetic, then on to algebra and plane geometry, farther along to solid geometry and trigonometry, and finally to calculus. At the same time, we'll point out to you the application of these subjects to your every-day affairs. That done, we plan to devote the remaining issues to a more complete consideration of the applications of mathematics to such fields as construction engineering, the machine shop, electricity, heat and chemistry, aviation and navigation, gunnery, etc.

The next issue will be along in about ten days, with articles showing you how to put to practical use the operations you've learned in this issue. We'll discuss decimals and their use in the practical affairs of life. There'll be a full treatment of averages, ratio, and proportion. In addition, we'll show you how to save yourself a great deal of time and energy through the use of logarithms and the slide rule to shorten your computations. There'll be some more of the "odd problems for off hours" to keep you and your friends guessing, and some more interesting side-lights on the strange ways numbers can be made to behave. Incidentally, the second issue will contain a list of the correct answers to all of the problems in this issue.

With the third issue, we shall turn our attention to algebra. In our presentation of that subject, Dr. Graves and I plan to show you how algebra may be utilized to probe the secret depths of many problems which cannot be solved with ordinary arithmetic. We have selected just those parts of algebra which are indicated in the prospectuses of the War and Navy Departments' training programs, together with the simple formulas needed in war industries.

The fourth issue extends your knowledge of algebraic principles. Dr. Dines and Dr. McGiffert will present some of the further principles of the subject for which you may be expected to have need. In this same issue, I shall have a short article on extracting square and cube roots, showing you that this process is not nearly so complicated and awesome as it is sometimes thought to be. Mr. Farr will show you how to make use of graphs in solving some types of algebraic problems.

With the fifth issue, we shall extend our concepts of mathematics to include geometry, the measurement of lines, surfaces, and solids. Dr. Bower's article on plane geometry will be supplemented by an article on simple construction exercises involving geometric principles.

In the sixth issue, Dr. Kasner will carry you into the field of solid geometry, including the construction and measurement of three-dimensional objects, such as prisms, cylinders, and spheres. You will get a glimpse of the possibilities of coordinate geometry as a means for determining the solutions to various problems which arise in daily work.

Dr. Agnew, in the seventh issue, will introduce you to the principles of trigonometry, showing you how to compute the values of angles and make use of known facts about angles in arriving at answers to various problems concerning length and distance.

By the time you reach the eighth issue, you will be ready for some work in the calculus. Dr. Wiener presents some of the simpler aspects of this subject, with direct applications to military and industrial problems.

To round out our theoretical consideration of mathematics, Dr. Menger will discuss the solution of differential equations and Dr. Baker will make a thorough investigation of measurement in the ninth issue.

Beginning with the tenth issue, we shall give you a series of articles on applied mathematics, making use of all of the theory which has gone before. While every issue of PRACTICAL MATHEMATICS will follow the pattern set up in this first issue, giving you problems which involve practical applications of each new subject presented, we believe that the work will not be entirely practical unless we round our your course by drawing together the various principles which apply to a given field.

In Issues Ten to Thirteen, we shall discuss successively machine-shop mathematics, construction engineering, heat and chemistry, electricity and radio, navigation and aviation, gunnery and ballistics.

Issue Number Fourteen, bringing this introductory course to a close, will present a review of the entire series, with general applications, and will contain an index to the whole course.

Meanwhile, off to a good start! Begin practising on the exercises and problems in this issue. Work for speed, but at the same time work for accuracy. You can set your own pace in this course, spending as long or as short a time on each problem as you think its worth merits in your own case. Once again, however, don't neglect the fundamentals!

R.S.K.

Basic Arithmetic

COURSE 1 **Practical Mathematics** PART 1

· WORKING WITH NUMBERS ·

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"WHY begin with 'simple arithmetic'? I learned all that years ago" may be your natural reaction as you begin this course. Most of us think that we are sufficiently familiar with "ordinary figuring"; we feel the urge to plunge immediately into more abstract matters. The experience of many men and women who have been called suddenly to new occupations in the armed forces and in industry shows, nevertheless, that it is well, at the outset of any study of mathematics, to make sure that the foundation is secure. Reports from training centers for candidates for the officers' schools of the United States Army and Navy indicate that those in charge of the training are calling for more attention to the basic operations. To that end, we begin this course on PRACTICAL MATHEMATICS with a consideration of the elements of arithmetic, which should serve as an introduction to the entire field. Mastery of these simple techniques will ensure greater facility at the more advanced stages.

THE LANGUAGE OF MATHEMATICS

Mathematics is an attempt to express known laws of relationship among numbers in concise, easily remembered form, to simplify their use. To that end, mathematics makes use of a sort of language or shorthand of its own. As we proceed with the various issues of PRACTICAL MATHEMATICS, the symbols will be introduced as they are needed and each will be explained in its proper place. To begin, we shall consider a few of the symbols which are in constant use in arithmetic.

The sign of equality

Almost every mathematical statement makes use of the word, *equals*. Instead of writing out this word every time that we have occasion to use it, we substitute instead the *sign of equality*, which is indicated by two horizontal bars, thus $=$. This sign is sometimes modified by being written with a slant line drawn through it, thus $\neq$, to signify "does not equal", or a question mark over it, thus $\stackrel{?}{=}$, to indicate "ought to equal". To indicate complete identity, this same sign is sometimes used with a third bar, thus $\equiv$, to mean "is identical with" or with a cur-

Fig. 1

ved line above, thus $\cong$, to be read "is congruent". This last meaning will be further explained when we come to the article on plane geometry which will appear in Issue Number Five.

The sign of addition

To indicate that two or more numbers are to be added, we write the numbers, joining them with the *plus* sign, $+$, which is usually read as "and" or "added to". ($2+2=4$, two and two are four, or two and two equal four.) In column addition, the sign is often omitted.

The sign of subtraction

To indicate that one number is to be subtracted from another, we use the *minus* sign, $-$, between the two numbers, placing first the number from which the other is to be subtracted, then the sign, and finally the number which is to be subtracted. ($8-5=3$, eight minus five equals three, or three from eight are five.)

Fig. 2

The signs of multiplication

Multiplication is indicated in several different ways. In arithmetic, the common practice is to join the two numbers which are to be multiplied by placing between them the sign, $\times$. ($6\times8=48$, six *times* eight are forty-eight.) This may also be indicated by using a dot in the center of the line between the two numbers, as $6\cdot8=48$. If we wish to show that one number is to be multiplied by several others, especially by their sum or difference, we may use parentheses to enclose the numbers which are to be combined before being multiplied, as $4(6+2)=32$, or four *times* the sum of six and two is thirty-two; $4(6-2)=16$, or four *times* the difference of six and two is sixteen. The operation indicated within the parenthesis is to be performed in each instance before the numbers are multiplied.

The signs of division

Division is usually indicated by the sign, $\div$. The expression, $8\div4=2$, is read "eight *divided by* four is two". This may also be written in the form, $\frac{4)8}{2}$ or $\frac{2}{4)8}$, the latter being particularly used in the case of "long division", which will be explained later (on page 20). Sometimes it is easier or more convenient to indicate the division in the form of a fraction, as $\frac{8}{4}=2$.

THE FUNDAMENTAL OPERATIONS

Arithmetical results are so important that we cannot afford to neglect any means of avoiding errors. Even in so simple a thing as adding numbers, one should have a definite method.

Since all calculation is based on the use of the nine Arabic numerals (and the zero, as explained on page 48), mastery of the combinations of these numbers is essential. At some time or other, everyone has learned what we come to regard as "the simple number facts". Many of us grow "rusty" on these, however, as time goes on; hence, it will be well to review with care simple combinations since all other arithmetical calculation depends upon an exact knowledge of these principles. We shall begin with the simplest, addition.

Addition

Addition is really a short way of counting. If we have three objects in one group and four in another, we may keep on counting, from the first object in one group to the last object in the other, and obtain the result, seven. As our groups grow larger, this process of counting may be found to be time-consuming. Observing that $4+3=7$ under all conditions, we adopt this statement and make use of the results without stopping each time to count to make sure that we have obtained the correct answer.

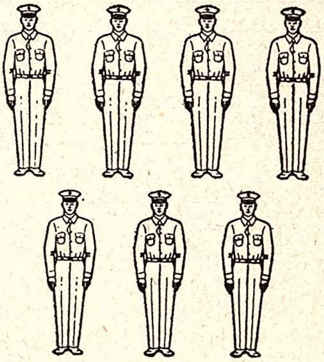


Fig. 3

So long as all the numbers which we are adding are less than 10, each will be expressed with a single *digit*, or figure.

If we have to add $3+4+2$, we may place them in a row, as in the line above, or in a column, as shown below. In column addition, we may add either up or down. In such cases, it is well to form the habit of performing such additions mentally, without saying all of the words, but simply calling the results, as shown in each instance beside the column.

Illustrative Example

(ADDING DOWN)		(ADDING UP)	
3	three	3	nine
4	seven	4	six
2	nine	2	two
9		9	

Before proceeding with more complicated exercises in addition, we shall find it advisable to master the simple addition facts shown in Table I (page 57), and then to extend our computations to include combinations of more than two numbers.

TEST YOUR ABILITY TO ADD DIGITS BY THESE EXERCISES

(Answers to all problems and exercises appearing in this issue will be printed in the following issue of PRACTICAL MATHEMATICS.)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
3	4	9	8	7	5	2	8
6	2	3	6	9	3	7	7
5	5	7	3	1	7	6	9
?	?	?	?	?	?	?	?

9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
8	7	3	7	9	8	5	2
2	1	6	3	4	4	3	8
5	4	8	2	3	2	7	9
3	9	2	8	6	7	6	7
?	?	?	?	?	?	?	?

ADDING TWO-DIGIT NUMBERS

When the numbers have more than one digit, we may begin with the right-hand column, adding it as above. We write the six under the units' column, "carrying" the one (which represents 10) to the tens' column and adding it to the numbers which appear there, then adding the second column in the same way.

Illustrative Example

	(SUM OF DIGITS IN RIGHT-HAND COLUMN)			(SUM OF DIGITS IN LEFT-HAND COLUMN)
		(*Carried*)	1	
17	seven		17	two
24	eleven		24	four
33	fourteen		33	seven
72	sixteen		72	fourteen
16			6	
			14	

This procedure may be repeated for any number of columns. After adding the left-hand column, we write the entire sum of that column, usually finding that the answer contains one more digit than the number of columns in the computation, as in the example above.

RULE FOR ADDITION

To add 1467, 985, 96, and 7834, set the numbers down, one under another, with the units' figures in a vertical column. Tens, hundreds, etc. will then fall in vertical columns in their proper places.

Begin at the top of the units' column and add down. Seven and five are twelve; twelve and six are eighteen; eighteen and four are twenty-two. We write two and "carry" two tens. (To save time, we should train ourselves to "think" the additions and simply say the results: 7, 12, 18, 22.) In the tens' column, beginning with the 2 we have carried, we have 2, 8, 16, 25, 28. Write 8 and carry 2. Then 2, 6, 15, 23. Write 3 and carry 2. Then 2, 3, 10. Write 10.

TEST YOUR KNOWLEDGE OF ADDITION BY THESE EXERCISES

17 In addition to learning the primary addition facts given in the table on page 57, you will find it advisable to memorize other combinations which arise when adding numbers in column addition or in carrying for multiplication. For this purpose, you may find it helpful to make use of the "number wheel" suggested in the accompanying illustration.

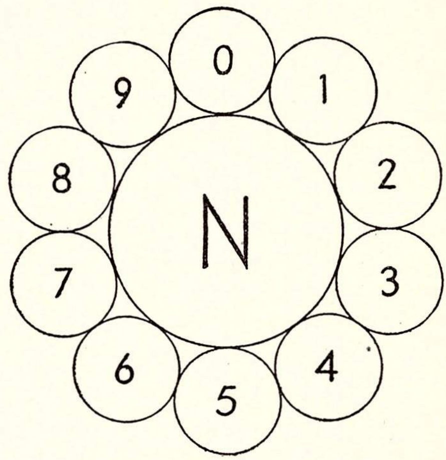


Fig. 4

Taking the diagram, substitute for the N in the middle each of the numbers from 10 to 39, inclusive, adding each in turn to each of the numbers shown in the small circles, as $10+0=10$, $10+1=11$, $10+2=12$, etc.; $11+0=11$, $11+1=12$, etc., up to $39+9=48$. Then take these numbers, which will also be needed in multiplication: 40, 42, 45, 48, 49, 54, 56, 63, 64, 72, 81, and add them around the circle in the same way. Mastery of these will give you 480 addition facts, which are all the facts that the ordinary person will need for any required addition in life. "Elementary!" you say? Yes, it is elementary, but it is also fundamental. Unless you have these combinations at your command for instant use, you will find yourself constantly slowed up in performing the computations which you will be needing at every step in advanced arithmetic. Simple as this exercise seems, it is the key to success in almost every mathematical step past this point. Mastery of addition makes much easier the work in subtraction, in multiplication, and in division—to say nothing of more involved figuring which will be encountered in the applied subjects later in the course. Incidentally, you will want to refer back to this diagram and perform similar drills in mental gymnastics for each of the other processes as you meet them in the later pages of this article.

Arrange in columns and add:

18 $34+22+43=?$ **20** $73+34+62=?$ **22** $83+76+59=?$
19 $52+43+31=?$ **21** $49+63+75=?$ **23** $64+45+93=?$

Arrange in columns and add:

24 $343+259+284=?$ **27** $689+224+386+758=?$
25 $573+767+962=?$ **28** $837+756+892+497=?$
26 $735+747+689=?$ **29** $271+959+326+924=?$

Add the following:

30 45, 689, 4, 7582, 17, 633, 5, and 8929.
31 179, 7696, 27, 859, 6, 8568, 205, 7069.

32		33		34		35		36	
	8967		8156		3432		3236		4453
	8651		9769		6259		2425		7257
	1572		3149		2449		7977		2226
	8941		8275		5216		4549		5878
	2035		2354		3431		8667		4892
	6174		3542		6428		6634		4662
	5184		8471		2268		3933		3787
	?		?		?		?		?

NOW APPLY YOUR KNOWLEDGE OF ADDITION TO THESE PROBLEMS

37 There are 19 telegraph poles on your side of the street, and 23 more on the other side. What is the number on both sides?

38 A dealer, after selling 24 gallons of gasoline to one customer and 36 gallons to another, had 75 gallons left. How many gallons had he at first?

39 A tank has two pipes. The first discharges 89 gallons a minute, the second 13 gallons more a minute than the first. How many gallons will both pipes discharge in a minute?

40 It is 152 miles from Nashville to Chattanooga, and Atlanta is 136 miles beyond Chattanooga. How far is it from Nashville to Atlanta?

41 After traveling 196 miles the first day and 216 miles the second day, a soldier hitch-hiking his way home was 248 miles from his point of destination. How far from the place of starting to his destination?

42 Texas has an area of 265,896 square miles; Oklahoma, 70,057 sq. mi.; Louisiana, 48,506 sq. mi.; Arkansas, 53,335 sq. mi.; Mississippi, 46,865 sq. mi.; Alabama, 51,998 sq. mi.; Tennessee, 42,022 sq. mi.; Kentucky, 20,968 sq. mi. What is the area of all the above states?

43 The population of Memphis in 1920 was 162,351; Atlanta, 200,616; Charleston, S. C., 67,957; Birmingham, 178,806; Jackson, Miss., 22,817; New Orleans, 387,219. What was the total population of these cities?

CHECKING CORRECTNESS

As a check, we begin at the bottom and add up: 4, 10, 15, 22, and so on. This is better than adding down a second time, as we are less likely to repeat an error previously made, since our combinations of numbers will fall in a different order.

In lengthy addition, it is sometimes more effective to write down the sums, rather than to carry, being sure that each result is placed in its proper column, with the figure being carried placed at the left, as in the second and third examples below.

	Adding Down	Adding Up
222 ← (*Carried*)	(*Begin at left*)	(*Begin at right*)
1467	1467	1467
985	985	985
96	96	96
7834	7834	7834
10382	8	22
	21	26
	26	21
	22	8
	10382	10382

Casting out 9's—A quicker check on the accuracy of an example in addition may be effected by learning the simple system of casting out 9's. To accomplish this, after having performed the addition as above, add the digits in each row of figures, subtracting 9 or any multiple of 9 which occurs, and write down the result. Do the same for each row and for the sum. Adding the resultants for the various rows should give a figure equal to the resultant for the sum.

	Digits		Resultant
1467	$1+4+6+7=18$	$(18-18=0)$	0
985	$9+8+5=22$	$(22-18=4)$	4
96	$9+6=15$	$(15-9=6)$	6
7834	$7+8+3+4=22$	$(22-18=4)$	4
10382	$1+3+8+2=14$	$(14-9=5)$	14 $(14-9=5)$

The sum of the resultants being 5 and the resultant of the sum also being 5, it is fairly safe to assume that the answer, 10382, is probably correct.

There are two ways of shortening this checking process. In the first place, as it is evident that every 9 which appears must eventually be cast out, the 9's may be ignored in the addition. Likewise, combinations of digits which add up to 9 (as 6 and 3, or 4 and 5) may be ignored when computing the resultants. In the given example, the problem would appear thus:

	Sum of Digits		Resultant
1467	$1+4+6+7=18$	$(18-18=0)$	0
985	(Ignore 9) $8+5=13$	$(13-9=4)$	4
96	(Ignore 9) Write 6 directly.		6
7834	$7+8+3+4=22$	$(22-18=4)$	4
10382	(Ignore $8+1=9$) $3+2=5$		14 $(14-9=5)$

A still more useful adaptation of this scheme makes use of a peculiar property of the number, 9. It will be found by observation that, if the digits of the first resultant are added, their sum will be the same as the resultant taken when multiples of 9 are subtracted. Hence, most people using this system of checking prefer to keep adding the digits together until they have resolved them to a single digit, as:

	First Resultant	Second Resultant	
1467	$1+4+6+7=18$	$1+8=9$ $(9-9=0)$	0
985	$8+5=13$	$1+3=4$	4
96	$6=6$		6
7834	$7+8+3+4=22$	$2+2=4$	4
10382	$3+2=5$ (or $1+3+8+2=14$)	$1+4=5$	14 $(1+4=5)$

Unfortunately, this test of checking accuracy by casting out 9's is not absolutely infallible. It will seem to indicate accuracy in cases where figures have been transposed or in cases where a 9 or any multiple of 9 is omitted.

For that reason, a more accurate system of checking is to be preferred. The following, although seemingly a bit more cumbersome, is recommended as a means of avoiding the possible errors pointed out in the preceding paragraph.

Casting out 11's—While the process of casting out 11's is slightly more difficult to perform, it has so much more accuracy that it is worth anyone's while to master it. In this scheme, the sum of the digits in the even-numbered columns is subtracted from the sum of the digits in the odd-numbered columns. The resultants are added just as in the case of the resultants obtained when casting out 9's. If the sum of the even-numbered digits is greater than the sum of the odd-numbered digits, 11 or some multiple of 11 is added to the sum of the odd-numbered digits to make that number greater than the sum of the even-numbered digits. When the result is obtained, 11 or the greatest multiple of 11 in the result is subtracted.

	Sum of Odd-numbered Digits	Sum of Even-numbered Digits	Resultant
1467	$7+4=11$	$6+1=7$	$11-7=4$
985	$5+9=14$	8	$14-8=6$
96	6 (add 11) $=17$	9	$17-9=8$
7834	$4+8=12$	$3+7=10$	$12-10=2$
			$\overline{20}$ $(20-11=9)$
10382	$1+3+2=6$ (add 11)	$0+8=8$	$17-8=9$

TEST YOUR KNOWLEDGE OF CHECKING ADDITION BY THESE EXERCISES

44 Using the examples done on page 6, check the results by casting out 9's.
45 Using some of the same examples or others of your own devising, check the results by casting out 11's.

MENTAL ADDITION

Instead of adding with pencil and paper, we frequently find it desirable to "think" the steps quickly. Since each number may be broken down into parts (as 1467 being thought of as composed of 1000, 400, 60, and 7), we may take the same problem on which we have been working in our exercises on addition and perform the additions mentally somewhat as follows:

1467	one thousand four hundred sixty-seven
985	and nine hundred are two thousand three hundred sixty-seven
	and eighty are two thousand four hundred forty-seven
	and five are two thousand four hundred fifty-two
96	and ninety are two thousand five hundred forty-two
	and six are two thousand five hundred forty-eight
7834	and seven thousand are nine thousand five hundred forty-eight
	and eight hundred are ten thousand three hundred forty-eight
	and thirty are ten thousand three hundred seventy-eight
	and four are ten thousand three hundred eighty-two
10382	

After a little practice, this method will be found quicker than it looks when set down in print. Omission of such words as "and" and "are", simply calling the numbers off mentally, as shown either at the left or the right of the figures below, is advisable.

one-four-six-seven	1467	fourteen sixty-seven
two-three-six-seven		twenty-three sixty-seven
two-four-four-seven	985	twenty-four forty-seven
two-four-five-two		twenty-four fifty-two
two-five-four-two	96	twenty-five forty-two
two-five-four-eight		twenty-five forty-eight
nine-five-four-eight		ninety-five forty-eight
ten-three-four-eight	7834	one-0-three forty-eight
ten-three-seven-eight		one-0-three seventy-eight
ten-three-eight-two		one-0-three eighty-two
	10382	

TEST YOUR ABILITY AT MENTAL ADDITION BY THIS EXERCISE

Using some of the examples previously given, try to add them mentally, following the suggestions given in the paragraph above.

Subtraction

After you have mastered the basic facts in addition, you are well on the way to having mastered subtraction as well. For instance, knowing that you get 7 when you add 4 and 3, you know instantly that 4 taken away from 7 leaves 3 or that 3 taken away from 7 leaves 4. It is well worth while to practice any combinations which give you difficulty in ordinary figuring, so that the harder combinations, like $16-9$, become just as automatic as $2-1$. All subtraction is based on the 100 combinations which give 9 or less for an answer, which are, in each instance, the reverse of one of the addition facts given in the table on page 57, and also on the answers to

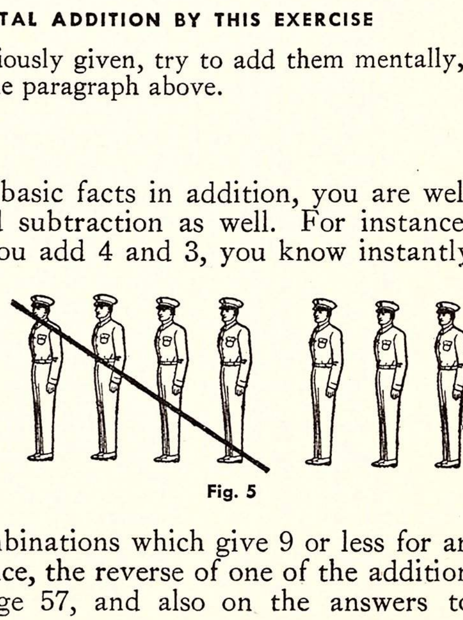


Fig. 5

175 related facts, in which one-digit results are obtained by subtracting one two-digit number from another (such as 37−29). You will find it to your advantage to prepare a subtraction table similar to the addition table just mentioned, and to drill yourself on any combinations which you find yourself constantly missing.

TEST YOUR ABILITY TO SUBTRACT BY THIS EXERCISE

46 Make a wheel similar to the one shown on page 5, except that the numbers shown in the outer circle are preceded by minus signs. Subtract each of these numbers from two-digit numbers chosen at random.

Subtract:

47	9797−1582=?	**50**	6867−834=?	**53**	11789−3776=?
48	4549−1230=?	**51**	7423−6420=?	**54**	17989−96=?
49	8859−1757=?	**52**	3696−2392=?	**55**	36843−9076=?

NOW APPLY YOUR KNOWLEDGE OF SUBTRACTION TO THESE PROBLEMS

56 At a naval training station with a personnel of 4,620 enlisted men, 2,417 were new recruits. How many had had some experience in the service?

57 A trader bought 600 head of cattle. He sold 250 and 10 died. How many had he left?

58 There were 3,250,475 soldiers engaged in a certain battle; 375,498 were captured or slain. How many were left?

A QUICK METHOD OF SUBTRACTION

If you are now able to subtract quickly and accurately, by all means stick to the method you are now using. If you are at all uncertain of accurate results, you might try this method. To subtract 37438 from 82076:

a Begin at the *left*, thus getting the most important figure right first. 8−3=5. Write one less than 5, because in the next column the 2 in the top row is less than the 7 underneath.

b 12−7=5. Write one less than 5, because in the next column the 0 in the top row is less than the 4 underneath.

c 10−4=6. Write 6, because the 7 in the top row of the next column is more than the 4 underneath.

d 7−3=4. Write one less than 4 because the 6 in the final column is less than the 8 underneath.

e Save the 1 to be combined (as a 10) with the 6, making 16. 16−8=8. Write 8.

$$\begin{array}{r} 82076 \\ -\underline{37438} \\ 44638 \end{array}$$

With a little practice, you will find that this method is quick and gives accurate results.

THE SHOP METHOD

There is also a method by which we may subtract several numbers at once. It is sometimes called the "shop method" because it is the method by which shopkeepers make up change. To subtract the numbers 2476, 87, 964, and 785, from 5063:

a Having four numbers to subtract, we add the negative numbers in the units' column upward: $6+7+4+5=22$. Since our minuend (the number from which the subtraction is to be made) ends in 3, we have to make the 22 up to the next number ending in 3, which is $22+1$, or 23. We write 1 as we say it and carry 2.

b This 2 we add in with the subtrahends (numbers to be subtracted) in the tens' column, as $2+7+8+6+8=31$. 31 and 5 make 36; write 5 and carry 3, to be added with the subtrahends in the third column.

c $3+4+9+7=23$. 23 and 7 make 30. Write 7 and carry 3.

d $3+2=5$. Since this number equals the number in the minuend, we leave the space blank. (A zero here would not change the meaning of the number; hence, we omit writing it.)

$$\begin{array}{r} 5063 \\ -\ \ 785 \\ -\ \ 964 \\ -\ \ \ 87 \\ -2476 \\ \hline 751 \end{array}$$

TEST YOUR ABILITY TO USE THE SHOP METHOD BY THESE EXERCISES

59 From 4798, subtract 642, 968, and 763.
60 From 15964, subtract 27, 864, and 936.
61 From 7494, subtract 85, 692, 809, 520, and 147.

CHECKING CORRECTNESS

There is an easy check on subtraction. When one number is subtracted from another, and the result written directly below, we add the two lower lines; the total should be the same as the top line. The check amounts to this: we have taken away a certain number; put it back and we have returned to the original number. The check should always be applied. It is said that "nine-tenths of the errors made in calculation are subtraction errors". Resolve to avoid them!

$$\begin{array}{r} 3402 \\ -1946 \\ \hline 1456 \end{array} \qquad \begin{array}{r} 3402 \\ -1456 \\ \hline 1946 \end{array} \qquad \begin{array}{r} 1946 \\ +1456 \\ \hline 3402 \end{array}$$

Subtracting the "answer" to your first problem from the original minuend should give you as a new answer the original subtrahend. Adding both numbers should give you the original minuend.

Subtraction, like addition, may be checked by casting out 9's or 11's. In this case, naturally, the resultant of the subtrahend is to be subtracted from the resultant of the minuend. The number thus obtained should check with the resultant of the answer. (If the resultant of the subtrahend is larger than the resultant of the minuend, add 11 to the resultant of the minuend before performing the subtraction.)

Casting out 9's

82076	(Ignore 7+2=9; add 8+6=14; 1+4=5; 5+9=14)	14
−37438	(8+3+4+7+3=25; 2+5=7)	− 7
44638	(Ignore 3+6=9; 8+4+4=16; 6+1=7)	7

Casting out 11's

	Sum of Odd-numbered Digits	Sum of Even-numbered Digits	Resultant
82076	6+0+8=14	7+2=9	14− 9=5
−37438	8+4+3=15	3+7=10	15−10=5
			0
44638	8+6+4=18 (18−11=7)	3+4=7	7− 7=0

TEST YOUR KNOWLEDGE OF CHECKING SUBTRACTION BY THESE EXERCISES

62 Using some of the examples on page 10 (or others of your own devising), check the results by casting out 9's.

63 For some other examples, check the results by casting out 11's.

Multiplication

Let us make sure that we know exactly what we are doing when we multiply. Suppose we want to find the number of men in 764 files, each of which contains 4 men; that is, 764×4. We can set down 764 four times and add. We can say:

a 4, 8, 12, 16; write down 6 and carry 1.
b 6, 12, 18, 24, and 1 are 25; write 5 and carry 2.
c 7, 14, 21, 28, and 2 are 30; write 30.

$$\begin{array}{r} 764 \\ +764 \\ +764 \\ +764 \\ \hline 3056 \end{array} \qquad \begin{array}{r} 764 \\ \times\ 4 \\ \hline 3056 \end{array}$$

On the other hand, we may multiply each of the digits in 764 by 4.

a Four 4's are 16; write 6 and carry 1.
b Four 6's are 24 and 1 are 25; write 5 and carry 2.
c Four 7's are 28 and 2 are 30; write 30.

That is, there is no great difference between adding and multiplying, except for the convenience and greater speed of the latter.

WHAT MULTIPLICATION IS

Now let us see what we do when we multiply a number which has several digits by another number which has several digits. To multiply 587 by 346, for example, it would be rather inconvenient to set down 587 to be added for a total of 346 times. Instead, we may consider that multiplying by 346 is the same as multiplying by 300 and by 40 and by 6, since $346=300+40+6$. To multiply by 6 gives no great difficulty, if we have mastered the simple multiplication combinations.

a Six 7's are 42; write 2 and carry 4.
b Six 8's are 48 and 4 are 52; write 2 and carry 5.
c Six 5's are 30 and 5 are 35; write 35.
d To multiply by 40, write a zero in the units' column; then proceed to multiply 587 by 4.
e Four 7's are 28; write 8 and carry 2.
f Four 8's are 32 and 2 are 34; write 4 and carry 3.
g Four 5's are 20 and 3 are 23; write 23.
h In the same way, to multiply by 300, we write two zeros before beginning to multiply 587 by 3.
i Three 7's are 21; write 1 and carry 2.
j Three 8's are 24 and 2 are 26; write 6 and carry 2.
k Three 5's are 15 and 2 are 17; write 17.
l Then add the numbers which you have set down as the results of these three multiplications, getting 203102 for the result.

After a while, you gain such proficiency that you may omit the step of writing down the 0's, being careful to move the product one place to the left when multiplying by the digit in the tens' column and two places to the left when multiplying by the digit in the hundreds' column. Your reasoning is the same, but you save yourself the labor of writing the useless zeros.

$$\begin{array}{rl} 587 & \\ \times 346 & \\ \hline 3522 & (6\times 587) \\ 23480 & (40\times 587) \\ +176100 & (300\times 587) \\ \hline 203102 & (346\times 587) \end{array} \qquad\qquad \begin{array}{r} 587 \\ \times 346 \\ \hline 3522 \\ 2348 \\ +1761 \\ \hline 203102 \end{array}$$

It is easy to see that three 4's are the same as four 3's. If we arrange 3 lines of 4 soldiers, we have three 4's. We have only to let the column wheel around or look at the men in columns instead of in rows in order to see them as four 3's. (Figure 6.) Now we may have any number of rows and any number of soldiers in a row.

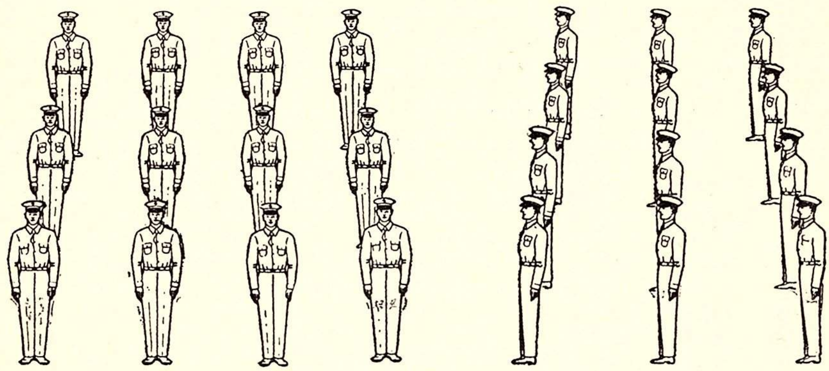


Fig. 6

Hence, we may say that *any number multiplied by any other number gives the same result as the second number multiplied by the first.*

This gives us a useful check on the accuracy of multiplication. We use the top number as the multiplier and the bottom number as the multiplicand. The result should, of course, be the same as before.

$$\begin{array}{r} 346 \\ \times 587 \\ \hline 2422 \\ 2768 \\ +1730 \\ \hline 203102 \end{array}$$

TEST YOUR KNOWLEDGE OF MULTIPLICATION BY THESE EXERCISES

64 Prepare a multiplication table, showing products of all numbers from 1×1 to 25×25, after the fashion of the table of addition shown on page 57. Retain this for future reference.

Multiply:

65 $\begin{array}{r} 63 \\ \times\ 3 \\ \hline ? \end{array}$ **66** $\begin{array}{r} 72 \\ \times\ 4 \\ \hline ? \end{array}$ **67** $\begin{array}{r} 91 \\ \times\ 8 \\ \hline ? \end{array}$ **68** $\begin{array}{r} 24 \\ \times\ 3 \\ \hline ? \end{array}$ **69** $\begin{array}{r} 143 \\ \times\ 2 \\ \hline ? \end{array}$

70 247 × 6 ?	73 1792 × 37 ?	76 4895 × 42 ?	79 1057 × 625 ?	82 40687 × 3409 ?
71 498 × 7 ?	74 4608 × 69 ?	77 1892 × 241 ?	80 4592 × 304 ?	83 91827 × 8685 ?
72 524 × 5 ?	75 7381 × 86 ?	78 2763 × 879 ?	81 67483 × 7962 ?	84 125863 × 7074 ?

NOW APPLY YOUR KNOWLEDGE OF MULTIPLICATION TO THESE PROBLEMS

85 If there are 69 tomato plants in each row of a victory garden, how many plants are there in 14 rows?

86 If there are 35 seats in a row, and 26 rows of seats in a room, find the number of seats there are in the room.

87 A post exchange sells 75 papers each day in the week. How many papers does it sell in 32 weeks?

88 If you bought $15 worth of war savings stamps a month for 27 months, how many dollars' worth of stamps would you have?

89 There are 16 gallon cans standing in a row. How will you find the number of pints of milk they contain?

90 A farmer sold 6 beef cattle whose average weight was 1150 pounds at 9¢ a pound. How much did he receive for them?

91 In the Government Printing Office, 500 large sheets of paper are cut into four parts. Each part is then cut into 7 pieces. Each piece is then divided into thirds. What is the resulting number of sheets of paper?

CHECKING CORRECTNESS

If the result is of great importance, we may use a further check. When two numbers are multiplied together, the result is called the product of the two numbers. If we divide the product by either of the numbers, we should obtain the other number. This check will be further explained in connection with the lesson on division, which follows on page 18.

Again, we may check by casting out 9's or 11's.

Casting out 9's

PROCESS	RESULTANT
346 (Ignore 6+3=9; resultant, 4)	4
587 (7+8+5=20; 2+0=2)	×2
203102 (2+0+1+3+0+2=8)	8

Casting out 11's

	Sum of Odd-numbered Digits	Sum of Even-numbered Digits	Resultant
346	6+3=9	4	9−4= 5
587	7+5=12	8	12−8= 4
			$\overline{20}$ (20−11=9)
203102	2+1+0=3 (3+11=14)	0+3+2=5	14−5=9

TEST YOUR KNOWLEDGE OF CHECKING MULTIPLICATION BY THESE EXERCISES

92 Using some of the examples on page 15 (or others of your own devising), check the results by casting out 9's.

93 For others of the examples on page 15, check the results by casting out 11's.

CONTINUED MULTIPLICATION

We sometimes want the continued product of three or more numbers; that is, we want to multiply the three numbers together. Thus, we might want to find $7 \times 8 \times 9$.

We may get the product of 7 and 8 and then multiply this by 9 to obtain the final product, as:

$$7 \times 8 = 56; \text{ then } 56 \times 9 = 504$$

For an easy check on such multiplication, we may simply reverse the order of the products, getting first the product of 8 and 9 and then multiplying that by 7 or the product of 7 and 9 and then multiplying that by 8.

$$7 \times 9 = 63; \text{ then } 63 \times 8 = 504$$

$$8 \times 9 = 72; \text{ then } 72 \times 7 = 504$$

In other words, it does not matter in what order the multiplication is performed, so long as all factors are included before the final result is obtained. This holds true no matter how many numbers are to be multiplied together. It is usually easier to group the obvious answers, performing the multiplication mentally, and then to set down the partial results and multiply them.

MULTIPLICATION SHORT-CUTS

To multiply by 10, we annex 0 at the right of the original number (thus moving the figure which was in the units' column to the tens' column, etc.). This is readily seen by performing some actual multiplications or additions. To multiply 96 by 10 (which would be the

same as adding ten 96's), we reason: since $96 = 90 + 6$, we may multiply 6 by 10 and 90 by 10 (or add 6 ten times and 90 ten times).

$$\begin{array}{r} 6 \\ 6 \\ 6 \\ 6 \\ 6 \\ 6 \\ 6 \\ 6 \\ 6 \\ \underline{6} \\ 60 \end{array} \qquad \begin{array}{r} 90 \\ 90 \\ 90 \\ 90 \\ 90 \\ 90 \\ 90 \\ 90 \\ 90 \\ \underline{90} \\ 900 \end{array} \qquad \begin{array}{r} 10\times 6 = 60 \\ 10\times 90 = \underline{900} \\ 10\times 96 = 960 \end{array}$$

Illustrative Problem

Suppose we want to find the number of cigarettes in 87 packs of 20 each; that is, 87×20. We find 10 times 87. Add two of these numbers and we get 20 times 87. It is easy to see that we get the same result by writing 0 under the end figure and multiplying by 2. Of course, we may use the same method for any number of tens, or for any multiple of 10, such as 100 or 1000, as is shown in the table on page 57.

$$\begin{array}{r} 87 \\ \underline{\times 20} \\ 1740 \end{array}$$

Now we may extend this idea to make possible quick multiplication by numbers other than 10, 100, 1000, and their multiples.

To multiply a number by 9 (and multiplying by 9 is one of the most difficult for some people because they are not too well acquainted with the multiples of 9), we might reason thus: since 9 is 1 less than 10, we might multiply by 10 and from that subtract the product of 1 times the original number (which, of course, is the same number). Thus, 468×9 would become $(468\times 10) - (468\times 1)$. Multiplying by 10 may be accomplished by annexing a zero. We set down 4680 and proceed to subtract 468.

$$\begin{array}{rr} (468\times 10) = & 4680 \\ -(468\times 9) = - & \underline{468} \\ & 4212 \end{array} \qquad \begin{array}{r} 468 \\ \underline{\times 9} \\ 72 \\ 54 \\ \underline{36} \\ 4212 \end{array}$$

In the same way, we may multiply by 8 by annexing a zero and subtracting twice the original number; or by 7 by annexing a zero and then subtracting three times the original number. For numbers just

less than 100 (such as 99, 98, or 97) or slightly under 1000 (such as 999, 998, 997), we may follow the same principles, annexing the proper number of zeros and subtracting the required number of times.

With a little practice, we may extend the idea a bit further, reasoning that 23 times a number is 25 times the number minus twice the number; that 124 times a number is 125 times a number minus the number, etc. Other similar extensions are obvious.

Division

Just as multiplication is a short form of addition, division may be considered a shorter method of subtraction. Furthermore, just as subtraction is the reverse of addition, so division is the reverse of multiplication. A clear understanding of these two statements will be of great help to you in putting division to use.

Illustrative Problem

Suppose we have a group of 639 pairs of socks. We want to issue them to 213 men so that each will get an equal share. How many pairs will each man receive? We might subtract 213 from 639 and keep subtracting 213 from the resulting answer until we finally reached zero. Counting the number of subtractions would indicate to us the number of pairs to be distributed to each man; in this case, 3.

$$\begin{array}{r} 639 \\ -213 \\ \hline 426 \\ -213 \\ \hline 213 \\ -213 \\ \hline 0 \end{array}$$

Obviously, this would be a slow and laborious method to use, particularly if the number of items were very large and the number of men in each group very small.

Fortunately, by making use of our knowledge that division is the opposite of multiplication, we may ask ourselves, "What number multiplied by 213 would give us 639?" From what we have already learned about numbers, we know that $213 = 200 + 10 + 3$ and $639 = 600 + 30 + 9$. We see readily that three times 200 would give us 600, three times 10 would give us 30, and three times 3 would give us 9; that is, that 3 times 213 would give us 639 $(600+30+9)$. The reverse would also be true, that 639 divided by 3 would give us 213.

$$\begin{array}{r} 3 \\ 213\overline{)639} \end{array} \qquad\qquad \begin{array}{r} 213 \\ 3\overline{)639} \end{array}$$

It is obvious, then, that a knowledge of the simple division steps will enable us to perform any step in division.

Let us take a slightly more difficult case. If we had 3056 men to be divided into four groups, we should run into difficulty if we broke the number up into parts, $3000+50+6$, since neither 50 nor 6 is exactly divisible by 4. In such a case, our reasoning would be:

a There are seven complete 4's in 30; or 700 complete 4's in 3000.
b Subtracting 2800 from 3056 leaves us 256 men whom we still want to count out.
c There are six complete 4's in 25, or 60 complete 4's in 256.
d Subtracting 240 from 256 leaves us still with 16 men to be counted out.
e There are four 4's in 16.
f Adding our quotients, we have $700+60+4=764$.

$$\begin{array}{rl} 764 & (700+60+4) \\ 4\overline{)3056} & \\ -\underline{2800} & (4\times 700) \\ 256 & \\ -\underline{240} & (4\times 60) \\ 16 & \\ -\underline{16} & (4\times 4) \\ 0 & \end{array}$$

Since we get "placement" value for our numbers when we write them in the proper columns, we may save ourselves some effort by shortening this process by omitting the 0's, just as we did in the case of multiplication, and by bringing down only the number or numbers which we are going to use in the next part of the example. It is only necessary to exercise reasonable care in placing the numbers directly under the columns in which they first appeared.

$$\begin{array}{r} 764 \\ 4\overline{)3056} \\ \underline{28}\downarrow\ \ \\ 25\ \ \\ \underline{24}\downarrow \\ 16 \\ \underline{16} \\ 0 \end{array}$$

When dividing with only one digit, we sometimes shorten the step still further by "thinking" the subtractions and "carrying" the remainders (often writing them in as is done in the following example), so that the whole problem is reduced in space and in the element of the time necessary to write it.

$$\begin{array}{r} 7\,6\,4 \\ 4\overline{)30{}^{2}5{}^{1}6} \end{array}$$

TEST YOUR KNOWLEDGE OF SHORT DIVISION BY THESE EXERCISES

94 Prepare a table of answers for short division (dividing by one digit) similar to the multiplication table you prepared in problem 64, page 14.

Divide:

95 $3\overline{)693}$	**99** $6\overline{)636}$	**103** $9\overline{)378}$	**107** $3\overline{)4392}$	**111** $6\overline{)72318}$
96 $4\overline{)848}$	**100** $8\overline{)432}$	**104** $6\overline{)738}$	**108** $5\overline{)4275}$	**112** $9\overline{)83709}$
97 $2\overline{)626}$	**101** $7\overline{)567}$	**105** $4\overline{)536}$	**109** $9\overline{)70101}$	**113** $8\overline{)10096}$
98 $2\overline{)636}$	**102** $5\overline{)425}$	**106** $7\overline{)224}$	**110** $8\overline{)30496}$	**114** $7\overline{)36505}$

LONG DIVISION

From these steps, the transition to long division is not difficult.

To divide 79872 by 624:

a Taking the first three digits of the dividend (since the divisor, 624, has three digits), we ask ourselves how many times 624 is contained in 798. Usually, we may employ the first figure in the divisor (here, 6) as a "trial divisor" to give us a clue to our answer.

b Our answer here being 1, we write the 1 over the *last figure* in the 798 on which we are working.

c Then we subtract the product from 798, just as in the examples above.

d We "bring down" the next number from the original dividend, forming a new dividend, 1747.

e Dividing 17 by 6, we decide upon 2 as the next digit in our quotient.

f Writing the 2 above the 7 in the original dividend, we multiply it by 624, getting 1248.

g Again subtracting, we have 499; again bringing down the 2, we form 4992, the new dividend.

h Dividing 48 by 6 would give 8; since this is nearest to the 49 with which the dividend begins, we decide upon 8 as the next figure in our quotient.

i Multiplying 624 by 8, we get 4992, which is exactly right, giving us a result of 0 after we perform the subtraction.

$$
\begin{array}{r l}
128 & \\
624\overline{)79872} & \\
\underline{624\downarrow\ \ } & (624\times1) \\
1747\ \ & \\
\underline{1248\downarrow} & (624\times2) \\
4992 & \\
\underline{4992} & (624\times8) \\
0 &
\end{array}
$$

Sometimes our problem will be such that we do not come out with a zero when we have brought down the digit from the units' column. In that case, we may express the remainder in the form of a fraction,

the remainder being written above the fraction-bar and the divisor below the bar, as in the example at the end of this paragraph. For more exact results, we sometimes place a decimal point, annex zeros, and continue the process of division, as will be explained in more detail in the lesson on decimals in Issue Number Two.

$$\begin{array}{rl}
 & 24\frac{183}{738} \\
738) & \overline{17895} \\
 & \underline{1476} \quad (738\times2) \\
 & 3135 \\
 & \underline{2952} \quad (738\times4) \\
 & 183 \quad \text{(remainder)}
\end{array}$$

Trial divisors—Ordinarily, it is sufficient to use the first digit in the divisor as the trial divisor in determining the number of times the entire divisor will be contained in the dividend. When the second digit is 5 or greater, however, it is usually safer to take as the trial divisor a number one higher than the first digit (as, in the case of 862, using 9 instead of 8 as the trial divisor). This can readily be appreciated by considering the example below. In the first instance, either 8 or 9 is contained in 20 twice. However, while 8 is contained in the 32 of the next divisor 4 times, it is obvious that 862 is contained in 3247 only 3 times; dividing 32 by 9 gives us this fact instantly. Again, 8 is contained in the 66 of the third divisor 8 times, but 862 is contained in 6611 only 7 times, which is the number obtained when 66 is divided by 9. Making use of this principle will eliminate a great many instances of re-multiplication which would be necessary if the 8 had been used throughout as the trial divisor.

$$\begin{array}{rl}
 & 237\ \frac{574}{862} \\
862) & \overline{204871} \\
 & \underline{1724} \\
 & 3247 \\
 & \underline{2586} \\
 & 6611 \\
 & \underline{6034} \\
 & 574
\end{array}$$

TEST YOUR KNOWLEDGE OF LONG DIVISION BY THESE EXERCISES

Divide:

115 $33\overline{)693}$	**118** $68\overline{)794}$	**121** $136\overline{)6042}$	**124** $208\overline{)61752}$
116 $47\overline{)9541}$	**119** $39\overline{)6243}$	**122** $545\overline{)70371}$	**125** $428\overline{)50002}$
117 $27\overline{)5481}$	**120** $78\overline{)5674}$	**123** $619\overline{)68435}$	**126** $961\overline{)12874}$

To save time in estimating whether or not a number is divisible by the trial divisor, one should memorize the rules for testing the divisibility of numbers, which are summarized in tabular form on page 58.

SHORT-CUTS IN DIVISION

At this point, it will be well to read again the instructions for short-cuts in multiplication (page 16). Since division is the reverse of multiplication, it is evident that these same steps may be taken, in reverse, to provide short-cuts in division. When we wish to divide by 10, we simply move the decimal point one place to the left; to divide by 100, move the decimal point two places to the left, etc. To divide by a multiple of 10, such as 20 (2×10), having moved the decimal point, we have only to divide by 2. The table on page 58 gives the directions for the more common steps which are required. Other combinations will readily occur to you after you have acquired facility in these. This is a matter which requires practice.

Examples

$380\div10=38.$	$380\div20=38\div2=19.$
$39.6\div10=3.96.$	$39.6\div30=1.32.$
$4200\div100=42.$	$4200\div300=42\div3=14.$
$1476.3\div100=14.763.$	$1476.3\div700=2.109.$
$2.74\div10=0.274$	$2.74\div20=0.137$
$9.636\div100=0.09639$	$9.636\div600=0.01606$

CHECKING CORRECTNESS

The process of division may be checked by multiplying together the divisor and the quotient. If these equal the original dividend, it is obvious that the division has been performed correctly. Hence, having divided 79872 by 624 and obtained 128 as the quotient, we next multiply 624 by 128. The product, being 79872, our original dividend, indicates to us that we have made no error in our division.

We may again make use of the process of casting out 9's or 11's, in this case dividing the resultant of the dividend by the resultant of the divisor and comparing the quotient thus obtained with the resultant of the quotient.

Casting out 9's

624	$6+2+4=12;\ 1+2=3$	3
$\times128$	(Ignore $1+8$) 2	$\times2$
79872	(Ignore 9 and $7+2$) $7+8=15;\ 1+5=6$	6

Casting out 11's

	Sum of Odd-numbered Digits	Sum of Even-numbered Digits	Resultant
624	4+6=10	2	10−2=8
×128	8+1=9	2	9−2=7
79872	2+8+7=17; 17−11=6	7+9=16; 16−11=5	6−5=1

NOW APPLY YOUR KNOWLEDGE OF DIVISION TO THESE PROBLEMS

127 If 36 men are divided into squads of nine, how many squads will there be?

128 How many hours will it take a truck to go 66 miles at the rate of 6 miles per hour?

129 There are 360 eggs in a crate. How will you find how many dozen there are?

130 At the conclusion of a U. S. O. party, the hostesses found that the 621 enlisted men who had been their guests had consumed 2484 doughnuts, 1242 cups of coffee, and 4347 cigarettes. Supposing that each man had received an equal amount, how many of each item did each man receive?

131 In how many hours will a tank holding 1480 gallons be filled by a pipe that pours into it at the rate of 185 gallons an hour?

132 A group of women collected 792 books to be sent to the Navy Library for the use of sailors on our warships. How many boxes will they need to ship them if 18 books will fit in one box?

133 The fuel tank of an airplane has a capacity of 207 gallons, and the airplane uses 23 gallons per hour at cruising speed. If the tank is full, how many hours can the plane stay aloft?

TEST YOUR KNOWLEDGE OF CHECKING DIVISION BY THESE EXERCISES

134 Using some of the examples on page 21 (or others of your own devising), check the results by casting our 9's.

135 For others of the examples on page 21, check the results by casting out 11's.

A FINAL CHECK-UP ON ARITHMETIC FUNDAMENTALS

136 A company of soldiers marched 40 miles in five days. The first day they marched 9 miles, the second day 10 miles, the third day 6 miles, and the fourth day 8 miles. How many miles did they march on the fifth day?

137 You know that there are 288 pages in a book. You also know that you can read 32 pages in an hour. How long will it take you to read the book?

138 A certain cattle dealer had 243 beef cattle, 477 sheep, and 49 lambs on hand. He sold 31 beef cattle, 83 sheep, and 9 lambs to private purchasers; the remainder he sold to the U. S. Army. How many animals of each kind did he sell to the Army?

139 In a certain army base, there are 16 tents, and 560 soldiers. What is the average number of soldiers to a tent?

140 A wiring job calls for the following amounts of insulated wire: mess

hall, 456 ft.; officers' club, 187 ft.; post exchange, 146 ft.; guard house, 125 ft.; three dormitories, 5,843 ft. How many feet of wire are needed for the job?

141 A certain war-production plant produces 2,547 torpedo cases on every eight-hour shift. If the plant operates on three shifts per day, how many torpedo cases does it produce in 31 days?

142 A broker bought 9 war bonds at $968 each, and sold them for $982 each. How much did he gain in the transaction?

143 An Army officer and his family moved from Chicago to St. Louis, a distance of 310 miles. They sent their furniture by moving van, which traveled at an average rate of 30 miles an hour. How long did it take the truck to make the trip?

144 Aluminum melts at 659°C. and boils at 1800°C. How many degrees below the boiling point is the melting point?

145 A U-boat makes 8 miles an hour under water and 15 miles an hour on the surface. How long will it take it to cross a 100-mile channel, if it has to go 40 miles under water?

146 A certain bomber can carry a bomb load of 4,500 lbs. How many 250-lb. bombs can it carry?

147 A circular sheet of aluminum is to be divided into sections of 18 degrees each. How many sections will there be when the circular sheet of metal is divided up? (A circle contains 360 degrees.)

148 A casting weighed 342 pounds. After machining, it weighed 297 pounds. How much of the casting was machined off?

149 A worker in a war-production plant who received $28 a week took special training in his particular field and was promoted to a position paying $149 a month. Counting 52 weeks in a year, how much more will he make a year in his new position?

150 A warehouse received an order from the Government for 790,800 eight-penny finishing nails to be delivered in one-pound packages. If there are 189 nails to the pound, how many cartons must the warehouse order?

151 A certain military bridge is 378 feet long, and is constructed of struts 14 feet in length. An enemy bomber destroys the bridge. How many 14-foot struts will be required to reconstruct the bridge?

152 At 60 cents an hour, how much can a workman earn in seventeen eight-hour days?

153 In the stock room of a war-production plant, there are 38 bins in each of 12 tiers. If each bin will hold 164 gears, what is the total capacity of the bins?

154 The area of the surface of the earth is 196,900,000 square miles. The water area is 139,750,000 square miles. How large is the land area?

155 In a tensile-strength test made on a testing machine, the tensile strength of a sample of steel wire was found to be 336,167 lbs. per sq. in., while the tensile strength of brass wire was only 148,239. How much stronger is the sample of steel wire than the sample of brass wire?

156 In 1940, the Navy appropriation for maintenance, public works, aircraft, and other expenses was $885,769,793; in 1941, the appropriation was $2,646,400,884; in 1942, the appropriation was $18,683,253,737; in 1943, the appropriation was $14,762,255,674. Find the total appropriations for these years.

COMMON FRACTIONS ·

By James McGiffert, Ph.D.

SO far in our discussion of operations with numbers, we have been dealing with whole numbers, or *integers*. Not infrequently, we find it necessary to deal with a part of a thing. Sometimes, we have special names for these parts, as when we say that an hour is divided into minutes or a foot into inches.

For many objects, however, we have no specific names for the parts. If we say, for example, that a pie is divided into pieces, we give no indication of the exact size of the pieces nor of the number of pieces into which the pie was divided. For any such division, whether it be pie or cake or a sheet of metal or a quantity of ore or a number of people divided into groups, we resort to fractional numbers to indicate what we have done or desire to do. (The word, *fraction*, comes from the same Latin root as the word, *fragment*, signifying a *part*.) In expressing fractional division, we understand that all of the parts into which the object has been divided are of exactly equal size.

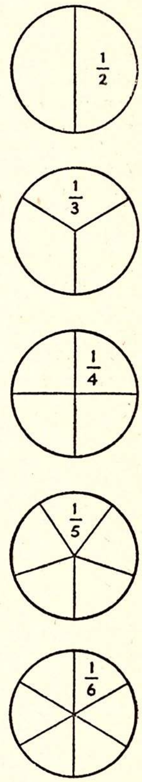


Fig. 7

NAMING PARTS OF FRACTIONS

A thing divided equally into two parts is said to be divided into *halves*. We express this division arithmetically by writing 1 (the unit) as the *numerator* of the fraction and the 2 (signifying the number of divisions) as the *denominator*, thus: $\frac{1}{2}$. We may interpret this as one divided into two parts. If divided into three equal parts, the object is said to be divided into *thirds*, each part being expressed as $\frac{1}{3}$. Similarly, an object divided into four equal parts is said to be divided into fourths, or *quarters*, the notation being $\frac{1}{4}$.

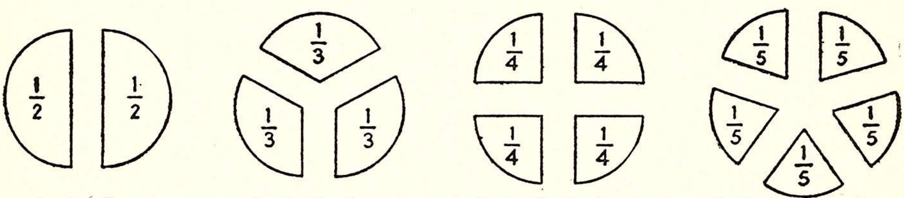
For any other division, we do not have to concern ourselves with special names, simply using the name for the number of divisions, with *-th* added, as one-fifth, $\frac{1}{5}$; one-sixth, $\frac{1}{6}$, etc.

Any fraction in which the numerator is smaller than the denominator, is known as a *common*, or proper, *fraction*. All of the fractions mentioned in the preceding paragraphs have been of this type; it does not follow that the numerator of a common fraction must be 1, however. Other examples of common fractions are $\frac{2}{3}, \frac{3}{4}, \frac{5}{7}$.

A foot-rule divided into inches is thus divided into twelve equal parts, each of which may be considered as one-twelfth $\left(\frac{1}{12}\right)$. The divisions of the inch are customarily marked out on the ruler in quarters, eighths, and sixteenths. If we want to express three-quarters, then, we take

$$\frac{1}{4}+\frac{1}{4}+\frac{1}{4} \text{ or } \frac{1+1+1}{4}=\frac{3}{4} \text{ of an inch.}$$

From this discussion, it should readily be observed that the two halves or the three thirds or the four fourths added together would again give us the whole unit on which we had been working, that is, that $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, etc. are all equal to one whole. In other words, any fractional expression of a number divided by itself, equals unity, as was observed in the table of divisions (page 20), when a number was divided by itself.



$\frac{1}{2}+\frac{1}{2}=\frac{2}{2}=1$ $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{3}{3}=1$ $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{4}{4}=1$ $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{5}{5}=1$

Fig. 8

Improper fractions

When we come upon an expression like $\frac{4}{3}$ (called an improper fraction because the numerator is larger than the denominator), this information helps us to see how we may treat it: $\frac{4}{3}$ may be considered as the sum of $\frac{3}{3}+\frac{1}{3}$; the $\frac{3}{3}$ is, as we have seen, equal to 1; therefore, $\frac{4}{3}$ equals $1\frac{1}{3}$. We may shorten the process by this simple rule: *Divide the numerator by the denominator; write the quotient as a whole number, followed by a fraction in which the remainder is expressed as numerator over the same denominator as before.* For example: $\frac{52}{15}=3\frac{7}{15}$, since 15 is contained in 52 three times, with a remainder of 7; $\frac{62}{25}=2\frac{12}{25}$, since

62 contains 25 twice, with a remainder of 12. An integer followed by a fraction is known as a *mixed number*. We shall consider first the treatment of common fractions, deferring until the latter part of the article (pages 35 to 37) the special treatments where mixed numbers are concerned.

Reducing fractions

When both numerator and denominator of the fraction contain a common factor, the fraction may be *reduced* by dividing both numerator and denominator by this factor. For instance, $\frac{15}{25}$ may be thought of as $\frac{3\times5}{5\times5}$. Since the $\frac{5}{5}$ equals 1, its removal would not change the value of the fraction and the fraction may be expressed as $\frac{3}{5}$. In final stages of computation, a fraction should always be reduced to its lowest terms (all like factors removed) and improper fractions should be expressed as mixed numbers.

OPERATIONS WITH FRACTIONS

With a few simple principles in mind, any of the four fundamental operations (addition, subtraction, multiplication, or division) may be performed with fractions almost as easily as with whole numbers. At this point, it may be well for you to refresh your memory as to the general principles underlying the fundamental operations (pages 1 to 24) before reading this section, in which the particular application of the operations to work with fractions will be discussed.

Multiplication of fractions

Perhaps the easiest of the four operations with fractions is multiplication. We have already seen that one-third of anything is $\frac{1}{3}$. Expressed arithmetically, this would appear: $\frac{1}{3}\times1=\frac{1}{3}$. In the same way, we have seen that $\frac{1}{4}$ taken three times is $\frac{3}{4}$, which would be expressed $\frac{1}{4}\times3=\frac{3}{4}$. From this, we may deduce the rule that, in multiplying a fraction by a whole number, we multiply the numerator of the fraction by the whole number, setting it over the original denominator.

To multiply a fraction by a fraction, let us take a simple illustration. In the illustration, the upper rectangle is divided into fourths. If we want to get one-third of one-fourth, we simply divide one of the sections of the upper rectangle into thirds. If we divide all of the

quarters into thirds, we find, by counting, that the figure is now divided into twelfths; that is,

$$\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}.$$

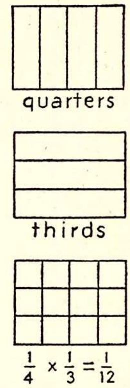


Fig. 9

From this, we see that, to multiply one fraction by another, we multiply the denominators of the two fractions. Actually, we have multiplied the numerators, also, as $1\times1=1$. The whole operation is more readily seen in this example:

$$\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}.$$

When mixed numbers are to be multiplied, change each to an improper fraction and proceed as above. Thus,

$$1\frac{2}{3}\times3\frac{1}{2}=\frac{5}{3}\times\frac{7}{2}=\frac{35}{6}=5\frac{5}{6}.$$

TEST YOUR ABILITY TO MULTIPLY FRACTIONS BY THESE EXERCISES

1 $4\times\frac{1}{5}=$?
2 $5\times\frac{1}{3}=$?
3 $6\times\frac{1}{2}=$?
4 $3\times\frac{1}{6}=$?
5 $2\times\frac{3}{10}=$?
6 $3\times\frac{5}{9}=$?
7 $4\times\frac{1}{6}=$?
8 $4\times\frac{5}{6}=$?
9 $\frac{1}{6}\times5=$?
10 $\frac{1}{5}\times6=$?
11 $\frac{1}{4}\times8=$?
12 $\frac{1}{4}\times2=$?
13 $\frac{3}{10}\times2=$?
14 $\frac{3}{10}\times4=$?
15 $\frac{1}{9}\times6=$?
16 $\frac{5}{9}\times6=$?
17 $\frac{1}{3}\times\frac{1}{5}=$?
18 $\frac{1}{3}\times\frac{4}{5}=$?
19 $\frac{2}{3}\times\frac{3}{4}=$?
20 $\frac{3}{5}\times\frac{5}{7}=$?
21 $\frac{1}{8}\times\frac{4}{5}=$?
22 $\frac{3}{5}\times\frac{10}{21}=$?
23 $\frac{2}{5}\times\frac{15}{16}=$?
24 $\frac{9}{10}\times\frac{6}{15}=$?
25 $\frac{6}{25}\times\frac{5}{9}=$?
26 $1\frac{1}{3}\times\frac{1}{4}=$?
27 $4\frac{1}{2}\times\frac{1}{4}=$?
28 $4\frac{1}{2}\times\frac{1}{3}=$?
29 $2\frac{1}{7}\times\frac{1}{3}=$?
30 $5\frac{1}{5}\times\frac{1}{2}=$?
31 $1\frac{1}{6}\times\frac{3}{5}=$?
32 $1\frac{5}{6}\times\frac{3}{5}=$?
33 $3\frac{3}{5}\times\frac{5}{6}=$?
34 $1\frac{1}{7}\times\frac{7}{12}=$?
35 $1\frac{7}{9}\times\frac{3}{5}=$?
36 $5\frac{5}{6}\times\frac{3}{5}=$?
37 $2\frac{1}{12}\times\frac{3}{10}=$?
38 $1\frac{1}{8}\times\frac{18}{25}=$?
39 $3\frac{3}{5}\times\frac{5}{12}=$?
40 $3\frac{3}{5}\times\frac{5}{9}=$?
41 $\frac{1}{2}\times1\frac{2}{5}=$?
42 $\frac{1}{3}\times3\frac{1}{2}=$?
43 $\frac{1}{5}\times2\frac{1}{2}=$?
44 $\frac{1}{5}\times3\frac{1}{3}=$?
45 $\frac{1}{3}\times2\frac{1}{4}=$?
46 $\frac{3}{5}\times1\frac{1}{2}=$?
47 $\frac{3}{5}\times2\frac{1}{2}=$?
48 $\frac{5}{6}\times2\frac{2}{5}=$?
49 $\frac{5}{8}\times1\frac{1}{5}=$?
50 $\frac{3}{5}\times1\frac{1}{9}=$?
51 $\frac{2}{5}\times2\frac{2}{9}=$?
52 $\frac{5}{8}\times3\frac{1}{15}=$?
53 $\frac{8}{15}\times1\frac{1}{4}=$?
54 $\frac{5}{12}\times1\frac{4}{5}=$?
55 $\frac{5}{6}\times1\frac{4}{5}=$?
56 $4\times1\frac{1}{5}=$?
57 $3\times1\frac{1}{3}=$?
58 $3\times1\frac{1}{6}=$?
59 $9\times1\frac{1}{6}=$?
60 $1\frac{1}{3}\times4=$?
61 $1\frac{1}{3}\times3=$?
62 $1\frac{1}{6}\times3=$?
63 $1\frac{1}{8}\times10=$?
64 $1\frac{1}{6}\times1\frac{2}{5}=$?
65 $1\frac{1}{2}\times2\frac{1}{3}=$?
66 $1\frac{1}{8}\times2\frac{1}{2}=$?
67 $1\frac{1}{7}\times1\frac{2}{5}=$?
68 $1\frac{1}{5}\times1\frac{1}{9}=$?
69 $1\frac{1}{6}\times2\frac{2}{3}=$?
70 $1\frac{2}{5}\times2\frac{1}{7}=$?

NOW TEST YOUR KNOWLEDGE OF MULTIPLYING FRACTIONS WITH THESE PROBLEMS

71 An author who had written 90 pages of a shop manual figured that $\frac{2}{3}$ of it was done. How many pages long is it likely to be?

72 The number of pages in a 96-page technical manual was increased $\frac{1}{6}$. How many pages were added?

73 An examination of 15 ladders in a repair shop showed that each should have 25 rungs. $\frac{1}{3}$ of the ladders had an average of 3 broken rungs, $\frac{1}{5}$ of the ladders had an average of 4 broken rungs. How many new rungs are required to replace the broken rungs?

74 The width of a door opening is $\frac{1}{4}$ of its height. What is the width if the height is $8\frac{2}{3}$ ft.?

75 The fuel tank of a plane holds 200 gals. If the gauge shows $\frac{5}{8}$ of the fuel has been used, how many gallons remain in the tank?

Division of fractions

When we divide a thing by 2, we take $\frac{1}{2}$ of it, as we have already seen on page 28. The process of dividing *by* a fraction is one of the most perplexing in the whole field of arithmetic. Let us approach it by a simple illustration. If we wish to convert a five-dollar bill into half dollars, we should obtain 10 fifty-cent pieces; that is, 5 divided into halves would give us 10 halves $\left(5\div\frac{1}{2}=10\right)$. Similarly, a two-dollar bill converted into quarters would give us 8 twenty-five-cent pieces $\left(2\div\frac{1}{4}=8\right)$. This is the same as multiplying 5 by 2 or 2 by 4. Since an integer may be thought of as a fraction with a denominator of 1 $\left(2=\frac{2}{1},\ 4=\frac{4}{1}\right)$, we may take as our rule: *To divide by a fraction, invert the fraction* (write the numerator as denominator and the denominator as numerator) *and multiply*. Thus,

$$\frac{2}{3}\div\frac{5}{7}=\frac{2}{3}\times\frac{7}{5}=\frac{14}{15}.$$

With mixed numbers, convert them to improper fractions, as in the case of multiplication, and proceed. Thus,

$$1\frac{1}{2}\div 2\frac{1}{5}=\frac{3}{2}\div\frac{11}{5}=\frac{3}{2}\times\frac{5}{11}=\frac{15}{22}.$$

CANCELING

Multiplication and division of fractions may frequently be simplified by performing certain divisions between numerators and denominators before proceeding with the indicated multiplications of numerators

or denominators. To multiply $\frac{3}{10}$ by $\frac{4}{9}$, instead of saying $\frac{3\times4}{10\times9}=\frac{12}{90}$ and then reducing by dividing both numerator and denominator by 6 (the common divisor of 12 and 90) to get $\frac{12\div6}{90\div6}=\frac{2}{15}$, we may obtain the result more directly by seeing that both the 3 and the 9 may be divided by 3 and both the 10 and the 4 by 2, then multiplying the resulting quotients, as:

$$\frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{10}}}\times\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}}=\frac{2}{15}$$

Again,
$$2\frac{5}{8}\div3\frac{3}{20}=\frac{21}{8}\div\frac{63}{20}=\frac{\overset{1}{\cancel{21}}}{\underset{2}{\cancel{8}}}\times\frac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{63}}}=\frac{5}{6}$$

TEST YOUR ABILITY TO DIVIDE FRACTIONS BY THESE EXERCISES

76 $\frac{1}{2}\div3=?$
77 $\frac{2}{5}\div7=?$
78 $\frac{2}{5}\div2=?$
79 $\frac{6}{7}\div3=?$
80 $\frac{3}{5}\div9=?$
81 $3\div\frac{1}{2}=?$
82 $3\div\frac{2}{3}=?$
83 $10\div\frac{2}{5}=?$
84 $3\div\frac{6}{11}=?$
85 $10\div\frac{15}{16}=?$
86 $\frac{1}{3}\div\frac{1}{2}=?$
87 $\frac{1}{2}\div\frac{1}{3}=?$
88 $\frac{1}{2}\div\frac{1}{6}=?$
89 $\frac{1}{6}\div\frac{1}{2}=?$
90 $\frac{5}{12}\div\frac{1}{2}=?$
91 $\frac{5}{6}\div\frac{1}{2}=?$
92 $\frac{1}{9}\div\frac{1}{6}=?$
93 $\frac{1}{6}\div\frac{1}{9}=?$
94 $\frac{1}{2}\div\frac{2}{3}=?$
95 $\frac{3}{10}\div\frac{2}{3}=?$
96 $\frac{1}{5}\div\frac{2}{5}=?$
97 $\frac{3}{4}\div\frac{1}{4}=?$
98 $\frac{1}{4}\div\frac{3}{4}=?$
99 $\frac{1}{6}\div\frac{3}{4}=?$
100 $\frac{3}{4}\div\frac{5}{6}=?$
101 $\frac{5}{8}\div\frac{5}{16}=?$
102 $\frac{5}{16}\div\frac{5}{8}=?$
103 $\frac{3}{8}\div\frac{3}{4}=?$
104 $\frac{8}{15}\div\frac{2}{5}=?$
105 $\frac{15}{16}\div\frac{9}{10}=?$
106 $\frac{9}{10}\div\frac{15}{16}=?$
107 $\frac{3}{8}\div\frac{3}{10}=?$
108 $\frac{3}{10}\div\frac{3}{8}=?$
109 $\frac{1}{4}\div1\frac{2}{3}=?$
110 $\frac{1}{5}\div2\frac{1}{2}=?$
111 $\frac{1}{3}\div1\frac{1}{5}=?$
112 $\frac{3}{5}\div1\frac{1}{3}=?$
113 $\frac{2}{5}\div1\frac{1}{5}=?$
114 $\frac{5}{7}\div2\frac{1}{10}=?$
115 $\frac{9}{16}\div5\frac{2}{5}=?$
116 $3\div2\frac{1}{6}=?$
117 $3\div1\frac{1}{6}=?$
118 $1\frac{1}{6}\div\frac{1}{6}=?$
119 $1\frac{1}{6}\div\frac{1}{3}=?$
120 $1\frac{1}{6}\div\frac{1}{9}=?$
121 $1\frac{1}{6}\div\frac{2}{3}=?$
122 $1\frac{1}{6}\div\frac{5}{12}=?$
123 $1\frac{1}{9}\div\frac{3}{7}=?$
124 $1\frac{1}{9}\div\frac{6}{7}=?$
125 $1\frac{1}{9}\div\frac{5}{6}=?$
126 $2\frac{7}{12}\div\frac{5}{8}=?$
127 $1\frac{1}{3}\div\frac{1}{4}=?$
128 $1\frac{1}{3}\div2=?$
129 $2\frac{1}{2}\div5=?$
130 $7\frac{1}{5}\div3=?$
131 $2\frac{2}{5}\div3=?$
132 $5\frac{1}{3}\div4=?$
133 $1\frac{4}{5}\div6=?$
134 $6\frac{3}{7}\div6=?$
135 $3\div2\frac{1}{4}=?$
136 $6\div1\frac{1}{3}=?$
137 $5\div2\frac{1}{2}=?$
138 $4\div3\frac{1}{5}=?$
139 $3\div1\frac{4}{5}=?$
140 $3\div3\frac{2}{5}=?$
141 $3\div1\frac{1}{5}=?$
142 $1\frac{1}{3}\div1\frac{1}{2}=?$
143 $1\frac{1}{2}\div1\frac{1}{3}=?$
144 $1\frac{1}{4}\div1\frac{3}{4}=?$
145 $1\frac{2}{5}\div1\frac{3}{5}=?$
146 $1\frac{3}{5}\div1\frac{2}{5}=?$
147 $1\frac{1}{4}\div1\frac{1}{5}=?$
148 $4\frac{1}{2}\div2\frac{1}{4}=?$
149 $2\frac{1}{4}\div4\frac{1}{2}=?$
150 $2\frac{1}{2}\div4\frac{1}{4}=?$
151 $4\frac{1}{4}\div2\frac{1}{2}=?$
152 $1\frac{1}{12}\div1\frac{1}{9}=?$
153 $1\frac{1}{9}\div1\frac{1}{12}=?$
154 $1\frac{2}{3}\div3\frac{1}{3}=?$
155 $3\frac{1}{3}\div1\frac{2}{3}=?$

NOW TEST YOUR KNOWLEDGE OF DIVIDING FRACTIONS WITH THESE PROBLEMS

156 The width of an aerial photograph is $\frac{2}{3}$ of its height. If the snapshot is enlarged so as to be $2\frac{1}{2}$ inches wide, how high is it?

157 How many sheets of metal, each $\frac{1}{16}''$ thick, are there in a stack 4′ 3″ high?

158 Using the distance around a circle as $\frac{22}{7}$ of the length of a line through the center, ending on the circle, find the thickness of a tree trunk which measures 110 in. around.

159 A soldier covers $1\frac{1}{2}$ acres with land mines in one day. How long will it take him to cover $8\frac{3}{4}$ acres?

Addition and subtraction of fractions

When we come to the problem of adding or subtracting fractions, we find ourselves faced with greater difficulty. So long as the denominators are the same, we add fractions just as readily as we add whole numbers, but, when the denominators are different, a new complication arises. This must be faced before we can proceed to perform the indicated addition.

FRACTIONS WITH LIKE DENOMINATORS

When we add 3 automobiles and 2 automobiles, we get 5 automobiles. Likewise, when we subtract 2 automobiles from 5 automobiles, we get 3 automobiles. Similarly, when we add 3 sevenths and 2 sevenths, we get 5 sevenths $\left(\frac{3}{7}+\frac{2}{7}=\frac{5}{7}\right)$; when we subtract 2 sevenths from 5 sevenths, we get 3 sevenths $\left(\frac{5}{7}-\frac{2}{7}=\frac{3}{7}\right)$. We may think of these processes, then, as being performed all over one denominator: $\frac{3+2}{7}=\frac{5}{7}$; $\frac{5-2}{7}=\frac{3}{7}$. Actually, we seldom bother to write out this step, but perform the combination mentally.

FRACTIONS WITH UNLIKE DENOMINATORS

When the fractions have unlike denominators, it is impossible to perform addition or subtraction until the denominators have been converted to like terms. In attempting to add 2 tables and 6 chairs, we still have 2 tables and 6 chairs; however, we may consider that we have 8 pieces of furniture; if we find 9 automobiles and 7 trucks in a garage, we may prefer, for brevity, to say that there are 16 motor vehicles housed therein.

If we want to add $\frac{1}{2}$ and $\frac{1}{3}$, we may resort to the same sort of reasoning. In the accompanying figure (on the following page), we start out with a rectangle which may represent our unit, whatever it

is. The second step shows the unit divided into halves. The third step shows it divided into thirds. In the fourth rectangle, the unit has been divided horizontally into halves and vertically into thirds. By counting, we observe that the rectangle has thus been divided into sixths. Comparing with figure 10, we see that $\frac{1}{2}$ equals $\frac{3}{6}$ and that $\frac{1}{3}$ equals $\frac{2}{6}$. Taking these equivalents, we may readily see that

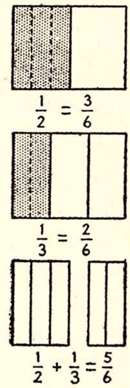


Fig. 10

$$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$$

or that

$$\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{1}{6}.$$

We may formulate the rule, then, that we obtain a common denominator for the two fractions by multiplying the two denominators together: $\frac{\cdot}{2}\times\frac{\cdot}{3}=\frac{\cdot}{6}$. To obtain the numerators of the transformed fractions, we divide the common denominator of the fractions by the original denominators, multiplying each numerator by the quotient thus obtained. In the case of $\frac{1}{2}$, 2 goes into 6 three times; 3 times 1 $\left(\text{the numerator of } \frac{1}{2}\right)$ equals 3, which is then written as the new numerator. Similarly, in the case of $\frac{1}{3}$, 3 goes into 6 twice; 2 times 1 $\left(\text{the numerator of } \frac{1}{3}\right)$ equals 2, which is then written as the new numerator.

To add $\frac{4}{7}$ and $\frac{2}{3}$, we arrive at the common denominator of 21 (7×3). We multiply 4, the numerator of the first fraction, by 3, the missing factor in the original denominator, getting 12. Again, we multiply 2, the numerator of the second fraction, by 7, the missing factor in its original denominator, getting 14 for the numerator. Thus,

$$\frac{4}{7}+\frac{2}{3}=\frac{4\times3}{7\times3}+\frac{2\times7}{3\times7}=\frac{12}{21}+\frac{14}{21}=\frac{26}{21}.$$

Since 26 is more than 21, we divide 21 into 26, getting for our final answer

$$\frac{26}{21}=1\frac{5}{21}$$

Lowest common denominator—While every necessary addition or subtraction can be performed by following the process set forth in the previous paragraph, the process may be shortened in many instances by determining first what factors the original denominators have in common and eliminating any duplications. In the case of $\frac{3}{4}+\frac{5}{6}+\frac{7}{10}$, we find that there is a common factor of 2. Taking this together with the unlike factors gives us, not $4\times6\times10$, or 240, but $2\times(2\times3\times5)$, or 60.

	Like	Unlike
4	2	2
6	2	3
10	2	5

Dividing this by 4 gives us as the multiplier of our first numerator 15; dividing by 6, we get 10 as the multiplier of the second numerator; dividing by 10, we get 6 as the multiplier of the third numerator.

$$\frac{3}{4}+\frac{5}{6}+\frac{7}{10}=\frac{3\times15}{4\times15}+\frac{5\times10}{6\times10}+\frac{7\times6}{10\times6}=\frac{45}{60}+\frac{50}{60}+\frac{42}{60}=\frac{137}{60}=2\frac{17}{60}.$$

If there is no common factor throughout the series, we modify our rule slightly: take each factor the greatest number of times it occurs in any one denominator. In the case of $\frac{3}{10}+\frac{4}{15}+\frac{7}{12}$, we first factor the given denominators. Starting with 12, we find the factors, 2, 2, and 3, all of which we shall use. Turning to 15, we find 3 (which has already appeared in 12) and 5, which we must use in the common denominator. Turning next to 10, we find that both factors, 2 and 5, have already been used. Our common denominator, then, will be $2\times2\times3\times5$, or 60. We next multiply each fraction by the missing factors: 10 being contained in 60 6 times, we multiply the fraction with a denominator of 10 by $\frac{6}{6}$, etc.

$$\frac{3}{10}=\frac{3}{2\times5}=\frac{3\times6}{10\times6}=\frac{18}{60}$$

$$\frac{4}{15}=\frac{4}{3\times5}=\frac{4\times4}{15\times4}=\frac{16}{60}$$

$$\frac{7}{12}=\frac{7}{2\times2\times3}=\frac{7\times5}{12\times5}=\frac{35}{60}$$

Where the fractions contain unusual numbers which are "hard" to factor because we do not have occasion to factor them very often, we find a table of factors (page 59) of considerable value to us in this process of determining the lowest common denominator.

TEST YOUR ABILITY TO ADD FRACTIONS BY THESE EXERCISES

160	$\frac{1}{5}+\frac{1}{5}=?$	171	$1\frac{3}{5}+2\frac{3}{5}=?$	183	$\frac{5}{6}+2\frac{1}{2}=?$	195	$\frac{1}{8}+\frac{1}{10}=?$
161	$\frac{1}{6}+\frac{1}{6}=?$	172	$1\frac{2}{5}+2\frac{3}{5}=?$	184	$1\frac{2}{5}+1\frac{1}{5}=?$	196	$\frac{1}{2}+\frac{1}{9}=?$
162	$\frac{3}{5}+\frac{3}{5}=?$	173	$1\frac{3}{10}+2\frac{3}{10}=?$	185	$1\frac{2}{3}+1\frac{4}{15}=?$	197	$\frac{1}{8}+\frac{9}{10}=?$
163	$\frac{5}{6}+\frac{5}{6}=?$	174	$1+\frac{1}{2}=?$	186	$2\frac{1}{2}+1\frac{3}{4}=?$	198	$\frac{1}{3}+1\frac{1}{4}=?$
164	$\frac{1}{5}+1\frac{1}{5}=?$	175	$4+3\frac{1}{2}=?$	187	$1\frac{1}{2}+1\frac{1}{18}=?$	199	$1\frac{1}{10}+\frac{1}{8}=?$
165	$\frac{1}{4}+1\frac{1}{4}=?$	176	$\frac{1}{10}+\frac{1}{5}=?$	188	$\frac{1}{3}+\frac{1}{2}=?$	200	$1\frac{1}{9}+\frac{1}{4}=?$
166	$\frac{2}{3}+1\frac{2}{3}=?$	177	$\frac{1}{6}+\frac{1}{2}=?$	189	$\frac{1}{2}+\frac{3}{5}=?$	201	$1\frac{1}{8}+\frac{3}{5}=?$
167	$\frac{2}{3}+1\frac{1}{3}=?$	178	$\frac{2}{3}+\frac{5}{6}=?$	190	$1\frac{1}{7}+\frac{1}{2}=?$	202	$2\frac{1}{6}+1\frac{1}{4}=?$
168	$\frac{5}{6}+1\frac{5}{6}=?$	179	$\frac{3}{5}+\frac{9}{10}=?$	191	$\frac{1}{5}+2\frac{1}{2}=?$	203	$1\frac{1}{4}+1\frac{1}{10}=?$
169	$1\frac{1}{5}+2\frac{1}{5}=?$	180	$\frac{1}{4}+1\frac{1}{2}=?$	192	$1\frac{1}{2}+2\frac{1}{3}=?$	204	$2\frac{1}{4}+1\frac{5}{6}=?$
170	$1\frac{1}{6}+2\frac{1}{6}=?$	181	$\frac{1}{4}+1\frac{1}{12}=?$	193	$2\frac{1}{2}+1\frac{2}{3}=?$	205	$1\frac{1}{8}+\frac{9}{10}=?$
		182	$\frac{3}{4}+1\frac{1}{2}=?$	194	$\frac{1}{4}+\frac{1}{3}=?$		

NOW TEST YOUR KNOWLEDGE OF ADDING FRACTIONS WITH THESE PROBLEMS

206 A pilot made four reconnaissance flights. The first one was $389\frac{3}{4}$ miles, the second $239\frac{2}{3}$ miles, the third $501\frac{1}{2}$ and the fourth $467\frac{5}{6}$ miles. What is the total distance he traveled in the four trips?

207 Determine the length of stock needed for the metal part shown in the accompanying figure. (Fig. 11.) (*Rule:* To find the length of stock needed, find the sum of the dimensions, and add $\frac{1}{2}$ the stock thickness for each bend.)

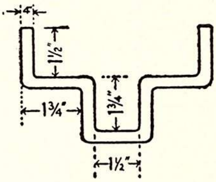


Fig. 11

208 A panel is made up of 5 plies which are $\frac{1}{4}$ in., $\frac{3}{8}$ in., $\frac{1}{3}$ in., $\frac{1}{5}$ in., and $\frac{2}{15}$ in. thick respectively. How thick is the panel?

209 On a blueprint, a crankshaft is indicated by the following dimensions: $3\frac{9''}{16}, 1\frac{3''}{8}, 7\frac{5''}{8}, 1\frac{3''}{8}, 6\frac{7''}{16}, 1\frac{3''}{8}, 7\frac{3''}{4}, 1\frac{7''}{8}$.
What is the total length of the crankshaft?

210 Four pieces of metal are to be welded together. They are $3\frac{1''}{2}$, $2\frac{2''}{3}$, $4\frac{5''}{6}$, $3\frac{7''}{8}$ long respectively. Allowing $\frac{1''}{8}$ for each joint, find the length of the finished piece.

TEST YOUR ABILITY TO SUBTRACT FRACTIONS BY THESE EXERCISES

211	$\frac{2}{5}-\frac{1}{5}=?$	218	$1\frac{3}{10}-1\frac{1}{10}=?$	225	$6\frac{1}{12}-\frac{7}{12}=?$	232	$1-\frac{1}{2}=?$
212	$\frac{5}{6}-\frac{1}{6}=?$	219	$3\frac{3}{5}-1\frac{1}{5}=?$	226	$7\frac{3}{5}-6\frac{4}{5}=?$	233	$7-\frac{1}{2}=?$
213	$\frac{1}{3}-\frac{1}{3}=?$	220	$5\frac{7}{8}-3\frac{3}{8}=?$	227	$3\frac{1}{6}-1\frac{5}{6}=?$	234	$5-3\frac{5}{9}=?$
214	$3\frac{2}{7}-\frac{1}{7}=?$	221	$2\frac{2}{3}-1\frac{2}{3}=?$	228	$8\frac{1}{3}-3\frac{2}{3}=?$	235	$4-2\frac{1}{2}=?$
215	$7\frac{3}{4}-\frac{1}{4}=?$	222	$1\frac{1}{4}-\frac{3}{4}=?$	229	$4\frac{1}{9}-2\frac{4}{9}=?$	236	$\frac{1}{3}-\frac{1}{6}=?$
216	$5\frac{2}{3}-\frac{2}{3}=?$	223	$1\frac{1}{6}-\frac{5}{6}=?$	230	$7\frac{1}{2}-7=?$	237	$\frac{1}{3}-\frac{1}{12}=?$
217	$1\frac{2}{3}-1\frac{1}{3}=?$	224	$4\frac{1}{5}-\frac{4}{5}=?$	231	$8\frac{1}{4}-5=?$	238	$2\frac{3}{4}-\frac{1}{2}=?$

239	$5\frac{5}{6}-\frac{1}{3}=?$	**248**	$8\frac{1}{4}-7\frac{5}{8}=?$	**257**	$6\frac{1}{2}-\frac{2}{5}=?$	**266**	$5\frac{1}{4}-4\frac{1}{6}=?$
240	$1\frac{5}{6}-1\frac{5}{12}=?$	**249**	$2\frac{1}{6}-1\frac{1}{2}=?$	**258**	$3\frac{1}{4}-2\frac{2}{3}=?$	**267**	$4\frac{1}{6}-3\frac{1}{10}=?$
241	$1\frac{9}{10}-1\frac{1}{2}=?$	**250**	$4\frac{1}{2}-1\frac{3}{4}=?$	**259**	$4\frac{1}{5}-2\frac{2}{3}=?$	**268**	$1\frac{1}{3}-\frac{4}{5}=?$
242	$4\frac{3}{4}-2\frac{1}{2}=?$	**251**	$4\frac{1}{6}-2\frac{2}{3}=?$	**260**	$\frac{1}{4}-\frac{1}{6}=?$	**269**	$1\frac{1}{6}-\frac{7}{10}=?$
243	$6\frac{3}{5}-2\frac{1}{10}=?$	**252**	$\frac{3}{5}-\frac{1}{3}=?$	**261**	$\frac{1}{8}-\frac{1}{10}=?$	**270**	$2\frac{1}{3}-\frac{3}{5}=?$
244	$1\frac{1}{12}-\frac{7}{12}=?$	**253**	$3\frac{3}{7}-\frac{1}{2}=?$	**262**	$1\frac{1}{3}-\frac{1}{5}=?$	**271**	$2\frac{1}{8}-\frac{3}{10}=?$
245	$2\frac{1}{2}-\frac{7}{10}=?$	**254**	$5\frac{1}{2}-5\frac{1}{3}=?$	**263**	$4\frac{1}{6}-\frac{1}{10}=?$	**272**	$2\frac{1}{3}-1\frac{3}{5}=?$
246	$9\frac{1}{4}-\frac{5}{8}=?$	**255**	$3\frac{5}{7}-1\frac{3}{5}=?$	**264**	$3\frac{1}{6}-3\frac{1}{10}=?$	**273**	$2\frac{1}{8}-1\frac{3}{10}=?$
247	$5\frac{1}{6}-\frac{2}{3}=?$	**256**	$3\frac{1}{3}-\frac{1}{2}=?$	**265**	$2\frac{1}{3}-2\frac{1}{5}=?$	**274**	$3\frac{1}{3}-1\frac{3}{5}=?$

NOW TEST YOUR KNOWLEDGE OF SUBTRACTING FRACTIONS WITH THESE PROBLEMS

275 A millman reduced the thickness of a $\frac{3''}{4}$-board by $\frac{5''}{16}$. What was the thickness of the finished board?

276 From a rod $3'6\frac{1''}{2}$ long, $\frac{1}{4}$ is cut away. Of the remaining part, $\frac{1}{4}$ is to be threaded. Find the length of the un-threaded section.

277 A plastic washer used in electrical instruments has an outside diameter of $1\frac{1''}{2}$, and an inside diameter of $\frac{22''}{32}$. How wide is the washer? (Fig. 12.)

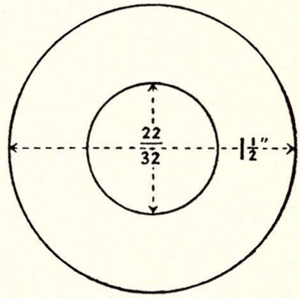


Fig. 12

278 A spindle $8\frac{15''}{16}$ is to be reduced to $6\frac{3''}{8}$. How much metal must be removed?

WORKING WITH MIXED NUMBERS

In working with numbers which contain both integers and fractions, we find ourselves faced with the necessity of combining several of the steps which have been discussed in the earlier sections of this article. While the process involves nothing new, we may find it desirable to review briefly the information that has already been given, in order that we may be a bit more sure of the procedure.

Addition of mixed numbers

In adding mixed numbers, we add together the integers just as in ordinary addition, and then the fractions (after changing all of the fractions involved to a common denominator). If the resulting fraction is an improper fraction, we reduce it to a whole number or a mixed number, adding the integral part to the integer previously found and appending the fractional part, if any. A few examples will aid in making the process clear.

Illustrative Problems

A
$$\begin{aligned} 27\tfrac{1}{6} &= 27\tfrac{3}{18} \\ 14\tfrac{2}{3} &= 14\tfrac{12}{18} \\ 9\tfrac{1}{9} &= \ \ 9\tfrac{2}{18} \\ \hline & \ \ 50\tfrac{17}{18} \end{aligned}$$

B
$$\begin{aligned} 123\tfrac{1}{4} &= 123\tfrac{2}{8} \\ 39\tfrac{3}{8} &= \ \ 39\tfrac{3}{8} \\ 275\tfrac{1}{2} &= 275\tfrac{4}{8} \\ \hline & \ \ 437\tfrac{9}{8} \end{aligned}$$

$$\frac{9}{8} = 1\frac{1}{8}$$

$$437 + 1\tfrac{1}{8} = 438\tfrac{1}{8}$$

C
$$\begin{aligned} 26\tfrac{3}{5} &= 26\tfrac{6}{10} \\ 7\tfrac{9}{10} &= 7\tfrac{9}{10} \\ 14\tfrac{1}{2} &= 14\tfrac{5}{10} \\ \hline & \ \ 47\tfrac{20}{10} \end{aligned}$$

$$\frac{20}{10} = 2$$

$$47 + 2 = 49$$

Subtraction of mixed numbers

When the fraction in the minuend is greater than the fraction in the subtrahend, we have no difficulty in performing subtraction. In such a case, we subtract the integer from the integer and the fraction from the fraction, arriving at our answer directly.

$$\begin{array}{r} 24\tfrac{4}{5} \\ -13\tfrac{1}{5} \\ \hline 11\tfrac{3}{5} \end{array}$$

If the upper fraction is the smaller of the two, we must first borrow one whole unit from the integer, converting it into the fractional equivalent of a whole number (as explained on page 26), add this to the fraction we already have, and then subtract the fraction of the subtrahend from the result.

Illustrative Problems

A
$$\begin{aligned} 47\tfrac{1}{4} &= 46 + 1\tfrac{1}{4} = 46\tfrac{5}{4} \\ -29\tfrac{3}{4} & \qquad\qquad\ = 29\tfrac{3}{4} \\ \hline & \qquad\qquad\ \ 17\tfrac{2}{4} = 17\tfrac{1}{2} \end{aligned}$$

B
$$\begin{aligned} 72\tfrac{1}{3} &= 72\tfrac{4}{12} = 71 + 1\tfrac{1}{12} = 71\tfrac{13}{12} \\ -37\tfrac{5}{12} & \qquad\qquad\qquad\qquad\ = 37\tfrac{5}{12} \\ \hline & \qquad\qquad\qquad\qquad\ \ 34\tfrac{8}{12} = 34\tfrac{2}{3} \end{aligned}$$

Multiplication and division of mixed numbers

In the case of multiplication or division, the rule is simple: *convert to improper fractions and multiply.*

Illustrative Problems

A $1\frac{1}{2}\times2\frac{1}{4}=\frac{3}{2}\times\frac{9}{4}=\frac{27}{8}=3\frac{3}{8}$

B $1\frac{1}{2}\div2\frac{1}{4}=\frac{3}{2}\div\frac{9}{4}=\frac{\cancel{3}^{1}}{\cancel{2}_{1}}\times\frac{\cancel{4}^{2}}{\cancel{9}_{3}}=\frac{2}{3}$

A FINAL CHECK-UP ON COMMON FRACTIONS

279 A carpenter can do one-half of a job in one day. A man training to do the same work can do one-fourth of the same job in one day. How much of the job can they do working together for one day?

280 A piece $17\frac{9}{64}''$ long is cut from a bar of metal $29\frac{31}{32}''$ long. What is the length of the remaining piece?

281 If it took a soldier 9 days to do one-third of a piece of work, how long will it take him to do all the work?

282 A machine bolt is $4\frac{1}{4}''$ long, the length of the cap is $\frac{5}{8}''$, and the threaded section is $\frac{7}{8}''$. How much of the machine bolt remains unthreaded? (Fig. 13.)

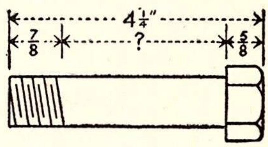


Fig. 13

283 How many pieces of iron can you cut from an iron bar 72″ long, each piece being $2\frac{1}{2}''$ long, allowing $2\frac{5}{8}''$ for loss in cutting?

284 A strip of steel 29 inches long is to be marked off into thirty-seconds of an inch. How many markings will there be?

285 How many gallons of alcohol will be required to fill 470 bottles, each holding $\frac{1}{5}$ of a gallon?

286 In a training station, one-third of the student body take aeronautical engineering courses, one-fifth take medical courses, one-sixth pre-flight courses, one-eighth ground school courses, and the remainder (420 students) take graduate flight training courses. How many students are taking courses?

287 The fuel allowance for an airplane on an operational flight is such that $\frac{5}{6}$ of it should be sufficient, the remainder being a safety margin. If an airplane takes off with 400 gallons in its tanks, how many gallons should be used if the operation is normal and none of the reserve fuel is required?

288 A dealer had 16 gallons of oil to sell. He sold $1\frac{1}{2}$ gallons to one customer,

$2\frac{3}{4}$ gallons to another, $7\frac{1}{4}$ gallons to another, and the remainder to the fourth customer. How much did he sell to the fourth customer?

289 A bit measures $2\frac{3}{8}''$ long. If a bit of this kind must be at least $1\frac{1}{4}''$ long to be usable, and if $\frac{5}{32}''$ are worn away every day when the bit is repointed, how many days may the bit be used?

290 A special nut used in radio instruments is $\frac{15}{16}''$ long; allowing $\frac{1}{4}''$ for the head, determine the length of the threaded section of the nut. (Fig. 14.)

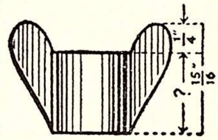


Fig. 14

291 Determine the total weight of 2,178,654 rivets in a Boeing Flying Fortress, assuming that the average weight of a rivet is $\frac{7}{16}$ of an ounce.

292 An alloy, used for bearings in machinery, is $\frac{24}{29}$ copper, $\frac{4}{29}$ tin, and $\frac{1}{29}$ zinc. How many pounds of each in 346 pounds of the alloy?

293 A plane flew at an average rate of $162\frac{2}{3}$ mph for $4\frac{1}{2}$ hrs. How far did it fly?

294 In a Vega Ventura, the length of a section of a wing channel is the sum of $\frac{7}{8}''$, $1\frac{1}{16}''$, and $\frac{9}{32}''$, another section is $3\frac{1}{4}''$. These two sections are joined by a third section, and the over-all length is $13\frac{1}{8}''$. Find the length of the third section in inches.

295 If a man can do a piece of work in 18 days, how much of it can he do (a) in 3 days, (b) in 12 days, (c) in 10 days?

296 From a barrel containing 74 gallons of oil, the toolroom custodian filled four containers holding, respectively, $\frac{3}{8}$ gal., $1\frac{1}{5}$ gal., $\frac{1}{2}$ gal., and $3\frac{3}{4}$ gal. How many gallons of oil remained in the barrel?

297 A length of electric cable in a war production plant is 15 feet long. What is left if worker A takes $3\frac{3}{8}$ feet, worker B takes $2\frac{7}{16}$ feet, and worker C takes $2\frac{1}{4}$ feet?

298 Seven Grumman TBF fighter planes (with a wing span of 51 feet) are parked tip to tip in formation with gaps of $3\frac{1}{4}$ feet between. Find the total length from the left tip of the first plane to the right tip of the seventh plane.

299 If sheet metal stock is $\frac{3}{64}''$ thick, how many sheets are there in a pile $3\frac{3}{4}''$ high?

300 The circumference of a circle is approximately $3\frac{1}{7}$ times the diameter. What is the circumference of a circle whose diameter is 19 feet?

SYSTEMS OF WEIGHTS AND MEASURES

By Robert N. Farr, Sc.M.

IT WOULD be difficult to imagine what this world would be like without our systems of measurement. Nearly everyone comes in contact with, and makes use of, weights and measures every day. We measure the *distance* from our homes to our places of work, the *speed* with which we get from home to work, and the *time* that it takes us. We buy our food and clothing by *weight*, *size*, *number*, or *length*.

THE PROCESS OF MEASUREMENT

Most of us are familiar with the process of measurement as applied to measuring a board. To measure a board, we simply lay down a foot-rule several times successively along the edge of the board. If we find that the board is 10 feet long, we know that this means the board is ten times as long as the foot-rule we used as our standard. A measurement may be said to involve two things, a *number* and a *unit*.

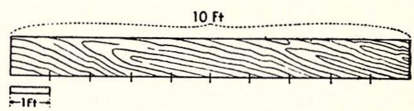


Fig. 15

Going one step further, we may say that to measure any quantity is to make a comparison with some other quantity of the same kind which is used as a standard. Thus, in the case of the board, we compared the length of the board with the standard of measurement, the foot-rule.

Since there are many different kinds of quantities to be measured, we must have appropriate standards and units for all of them. Whether we are concerned with measuring the length of a board, the speed of an airplane, the temperature of a room, the rating of an electric light bulb, or one of hundreds of other instances you can readily think of, we must employ one or more of the three fundamental units of measurement, length, time, mass (or weight). Through the ages, men have devised various means of designating these units, some of them highly functional, some of them rather complicated, and a few of them really convenient.

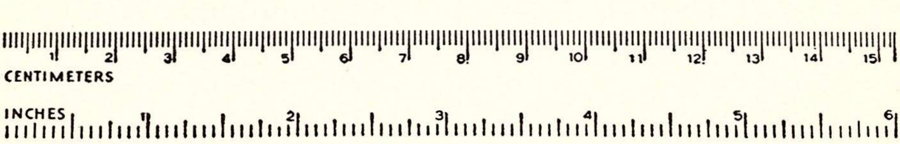


Fig. 16

SYSTEMS OF MEASUREMENT

There are two systems of measurement in general use today, the *metric* system, and the system with which we are most familiar, the *English* system (also referred to as the British system, or the American system). The metric system is standard in all countries, with the exception of the two great English-speaking nations, Great Britain and the United States. The metric system is also the international scientific and engineering standard of measurement, and as such it is used in all countries. It is important for us, therefore, to be able to use both the British and metric systems of weights and measures.

Standards for measuring length were taken from actual measurements of parts of the body. Fingers, feet, arms—all became units of length. Given different names in different countries, they nevertheless came to be recognized as being of approximately equal value.

The length of the forearm from elbow to the tip of the middle finger was the unit of measurement employed by the ancient Egyptians for the *cubit* (about 20 inches). At a later date, the Greeks developed the Olympic cubit, which, subdivided into 12 thumb-nail breadths, became the *foot*. The foot unit was borrowed from Greece by Rome, where each subdivision became known as an *uncia*, a word which has passed over into English as *inch*.

In mediæval Britain, it became customary for the standards of such units of measure to be determined by measuring parts of the body of a king. Since people vary in size, it is evident that such a "standardization" resulted in perplexing changes of units of measurement as the crown passed from head to head. A yard might be lengthened or shortened considerably in accordance with the actual measurement of the "stretch" of a given king. Confusion naturally resulted. Overnight, the measurement of a bolt of cloth might shrink or expand considerably, with consequent effects on the wealth of the owner.

The British system of measurement

In Britain, the concept of *foot* was merged with the Anglo-Saxon measures, which also contained the *fathom*, or length across the two arms outstretched, or 6 feet. As half-a-fathom was found to be a more convenient length, this was designated as the *yard*, the equivalent of 3 feet, and a bronze yard-bar was kept as the Standard of Reference in the King's Exchequer. When the settlers came to America, each group brought the weights and measures of the homeland to this country, and it was in this manner that the British System of Measurement was introduced here. Two copies of the British Standard Yard were presented to the United States in 1856. These copies were afterwards accepted by the Office of Weights and Measures as the standards for the United States.

The British system of measurement uses the *foot* as the unit of length, the *pound* as the unit of weight, and the *second* as the unit of time.

The Metric System

The metric system, which derived its name from its fundamental unit, the *meter*, was originated in France about 1800. All units of weights and measures are based upon the meter. The meter was determined by figuring one ten-millionth of the distance from the equator to the North Pole, along the meridian passing through Paris. Although a slight error was made in determining this standard unit, the usefulness of the system is in no way affected. In scientific work, the metric system is almost universally accepted because its use greatly reduces the work of making computations. The metric system has never been adopted by industry in the United States, except in instances where products are manufactured for export to foreign countries, where the metric system is in common use.

One of the advantages of the metric system of weights and measures is its simplicity. There are only five tables covering length, area, volume, cubic measure, and weight. There is only one scale, based upon the meter.

Another feature of the metric system which makes it extremely useful is that it is standard all over the world. A meter, for example, is the same length in Brazil as it is in the United States. Under the British system, there are many variations. A gallon of gasoline in the United States is about 20 per cent less than the Imperial gallon of gasoline in the British Commonwealth.

It is easy to understand the metric system because it is based upon decimals[1]. The standard unit of length is the *meter*. The standard unit of volume is the *liter*. The standard unit of weight is the *gram*. Latin and Greek prefixes are used to indicate subdivisions and multiples of the standard units. The Latin prefixes are used to indicate subdivisions, as follows:

$$\text{milli} = \frac{1}{1000} = \text{one thousandth}$$

$$\text{centi-} = \frac{1}{100} = \text{one hundredth}$$

$$\text{deci-} = \frac{1}{10} = \text{one tenth}$$

Thus, a milli-meter, milli-liter, or milli-gram mean $\frac{1}{1000}$ of a meter,

[1] Decimals are introduced in "Numbers through the Ages" on page 49 of this issue. A detailed presentation of the subject will appear in Issue Number Two of PRACTICAL MATHEMATICS.

liter, or gram; a centi-meter, centi-liter, or centi-gram means $\frac{1}{100}$ of a meter, liter, or gram; and a deci-meter, deci-liter, or deci-gram means $\frac{1}{10}$ of a meter, liter, or gram.

The Greek prefixes are used to denote multiples, as follows:

Deca-	=	10	=	ten
Hecto-	=	100	=	one hundred
Kilo-	=	1000	=	one thousand
Myria-	=	10000	=	ten thousand.

Thus, Deka-meter, Deka-liter, or Deka-gram means 10 meters, liters, or grams. Hekto-meter, Hekto-liter, or Hekto-gram means 100 meters, liters, or grams. Kilo-meter, Kilo-liter, or Kilo-gram means 1000 meters, liters, or grams. The same procedure holds for the other prefix, Myria.

Anyone familiar with our system of counting money need have no fear of the metric system. Our word, *dime*, meaning *ten cents*, and the word *decimal*, meaning *numbered by tens*, are both derived from the Latin word, *decem*, meaning *ten*. From our system of dollars and cents (which is based upon a decimal standard), we know that

10 cents	=	1 dime
10 dimes	=	1 dollar.

In using the metric system, we operate also on the basis of ten, using the three main words, *meter*, *liter*, and *gram*, modified by the Latin or Greek prefixes, depending upon the division or multiple; thus:

10 millimeters	=	1 centimeter
10 centimeters	=	1 decimeter
10 decimeters	=	1 meter.

Metric tables are formed by combining the words, meter, liter, gram, with the seven numerical prefixes as we have shown above. The metric tables on page 62 give divisions, multiples, and abbreviations of the most important metric units. An examination of these tables will show you that any unit in the metric system is ten times as large as the next smaller unit.

WEIGHTS AND MEASURES IN THE UNITED STATES

The Constitution of the United States gives to Congress the power to establish standards of weights and measures. The only legal standard that has been established by the Congress is the *troy pound* for the use of the mint. *Troy weight* is a

system of weights commonly used for gold and silver, having 12 ounces for its basic unit. Beyond this legislative action, our weights and measures in ordinary use are dictated by custom, with indirect Congressional recognition. Metric weights and measures were legalized in the United States in 1866 by direct legislative permission. Standards for both systems, made of platinum and iridium, are kept in the Office of Standards of Weights and Measures in Washington. To ensure accuracy, the copies of the metric standards in Washington are comprepared at intervals, in normal times, with the international standards kept at the Bureau of Weights and Measures near Paris.

Measures of length

In the United States, the fundamental standards for measuring length are the *inch*, the *foot*, the *yard*, and the *mile*. The basic unit of length in the metric system is the *meter*. The meter is equivalent to about 39.37 $\left(39\frac{37}{100}\right)$ inches, or a little over three feet. For measuring shorter lengths, the *centimeter*, $\frac{1}{100}$ of a meter, and the *millimeter*, $\frac{1}{1000}$ of a meter, are used. For longer lengths, the *kilometer*, 1,000 meters, is used. Fig. 16 (page 39) shows a rule with both metric and British scales. Upon a close examination of this scale, you will note that one inch equals a little more than two and one-half centimeters. If we were to carry this still further, we should see that one mile equals more than one and one-half kilometers.

Measures of area or surface

The area of a surface is the amount of space it occupies. The area of a rectangle, for example, is found by multiplying its length by its width. The fundamental standards for measuring area in the United States are the square inch, the square yard, and the acre. There is no fundamental standard of area measurement in the metric system. A comparison of the systems shows that 1 square inch is over six times as large as 1 square centimeter. In the metric system, 1 square meter equals more than 1 square yard. The square Dekameter and the square Hektometer, when used to measure land, are sometimes called the *are* and the *hektare*. One hektare is equal to about $2\frac{1}{2}$ acres.

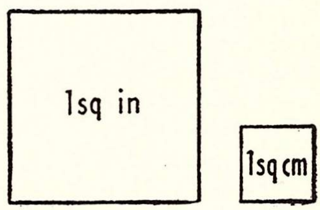


Fig. 17

Measure of volume and capacity

The fundamental standards for measuring volume in the United States are: (1) the cubes of linear units, such as the cubic inch, the cubic foot, and the cubic yard, used for measuring cubic contents; (2) the gallon, equal to 231 cubic inches, employed in liquid measure; (3) the bushel, equal to about $2150\frac{1}{2}$ cubic inches, used in dry measure.

The common metric measures for volume and capacity are: (1) the cubic centimeter, the cubic decimeter, and the cubic meter, used for measuring cubes of linear measurements, or volume; (2) the liter, which equals 1000 cubic centimeters, is the common unit of capacity. Since one cubic centimeter of water weighs 1 gram, and 1 liter equals 1000 cubic centimeters, we may conclude that 1 liter of water weighs 1 kilogram. A comparison of the metric and British units of volume shows us that 1 cubic inch equals a little more than 16 cubic centimeters, and 1 cubic yard equals about $\frac{3}{4}$ of a cubic meter. The cubic meter is called a *stere* when used in measuring wood. Considering units of capacity, we note that the liter (the principal unit of capacity) is equal to a little less than a dry quart. In relation to a liquid quart, the liter is slightly larger.

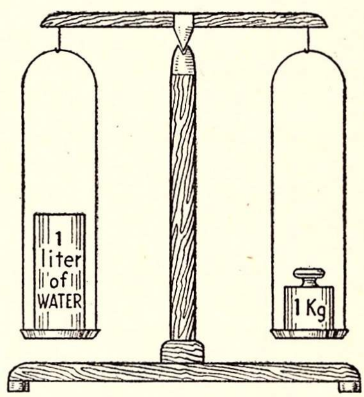


Fig. 18

Measures of weight

The fundamental standard of weight in the United States is the *pound.* The standard pound, as used in the United States is based on the kilogram, one pound being equal to 0.4536 kilogram. This value is the one established at the National Bureau of Standards in Washington. The other units in common use in the United States (grains, ounces, and tons) are also based upon the metric kilogram.

The *gram,* which is the primary unit of weight in the metric system, is the weight of 1 cubic centimeter of water at a certain temperature. The gram is used in weighing gold, jewels, and small amounts of certain articles. The kilogram, commonly termed the *kilo,* is used in measuring the weights of larger objects, and the metric ton (1000 kilograms) is used for measuring the weight of heavy articles.

The ton, which equals 2000 pounds, equals approximately $\frac{9}{10}$ of a metric ton.

NUMBERS THROUGH THE AGES

THE origin of our system of numbers is shrouded in mystery. Every race and every great civilization has contributed something important to the structure of our system of numbers. Man has many means of expression, but the language of numbers is probably the most universal.

Only a few thousand years ago, numbers were seldom needed. When a primitive man wished to show that he wanted ten coconuts, he would hold up both hands, showing ten fingers. This system, called "finger notation", is still in use today. On the floor of the Stock Exchange, sales of stocks and bonds are indicated by a system of finger notation.

EARLY NUMBER SYSTEMS

Primitive man could rarely count above ten, for he had only ten fingers. As time went on, however, it became necessary for him to devise a means of counting above ten. He would say, "Ten and one", or "Ten fruits and one fruit". This system of counting was probably the origin of the decimal system which is in international use today. We know that small children frequently make an attempt to indicate quantity by holding out the required number of fingers to show the number of objects which they desire to express. Many of us remember the reprimands which we received in primary school when the teacher caught us counting on our fingers. The abacus which the Chinese laundryman uses in computing the extent of your bill is an extension of this finger-counting.

As civilization advanced, the need for an exacting system of numbers became greater. The use of symbols was introduced in ancient Egypt and Babylonia. The Egyptians represented the numeral 1 by a vertical line, 10 by a horse-shoe, 100 by a corkscrew, 10,000 by a pointing finger, and 1,000,000 by a man with an astonished expression on his face. The Egyptians became so adept at figuring that their architects, who measured the base of the Great Pyramid of Gizeh, completed their work with an error in the sides of only $\frac{1}{27,000}$ of an inch.

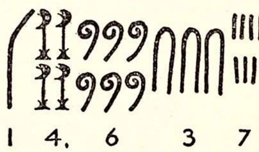


Fig. 19

As the Babylonians were fine astronomers, they needed units of measure for time and angular distances. They figured 360 days in the year, and 12 months each with 30 days, and they decided upon 60 as a base for their numbers. These people gave us the principle that determines 60 minutes in an hour, 60 seconds in a minute, and 360 degrees in a circle. The division of the year into 12 months was

probably suggested by the use of 12 hours for the day and 12 hours for the night. The analogy is easy to follow. The difficulties in the duodecimal system prevented its common acceptance, however.

The nine digits now in use can be traced back to the Arabs, who borrowed them from the Hindus. The Hindus and the Arabs provided us with a system of numbers that is indispensable in modern life. Calculations involved in the construction of modern machines, the airplane, the radio, the automobile, would be impossible without this system of numbers. In the preceding lessons on basic arithmetic and fractions, we have been using Arabic numerals. If we were to try to perform arithmetic operations with another system of numbers, such as the Roman system, we should soon realize how important a contribution to modern civilization the Arabic numeral really turned out to be. Addition or subtraction would not be absolutely impossible, but multiplication and division would prove themselves to be very taxing on one's mental powers, and would impede accurate computation.

Fig. 20

THE ROMAN SYSTEM

The Roman system of notation was inherited from the Etruscans. The Roman signs for the first three numbers, 1, 2, 3, are represented by vertical lines. Since the form, IIII, is rather difficult to read, the number, 4, was written thus: IV. To write 6, 7, and 8, the Romans added vertical lines to V, which is the figure used to represent 5, thus: VI, VII, and VIII. In addition to I and V, the Roman system of numbers incorporated other letters to represent numbers:

X	L	C	D	M
10	50	100	500	1000

Other numbers in the Roman system were represented by combinations of these letters. When a letter is repeated, its value is to be taken as many times as the letter appears, as, for example:

$$\begin{aligned} X &= 10 \\ XX &= 20 \end{aligned}$$

and so on. When a letter is followed by one of lesser value, the values are added; thus, VI = 6. If a letter is placed before another of greater value, it is to be subtracted; thus, IV = 4. In the same way, IX (one

from ten) represents nine, XL (ten from fifty) represents forty, and so on. A bar above a number increases its value 1000 times, as: $V=5$; $\overline{V}=5000$.

We can easily see how difficult it would be for us to perform simple multiplication by the use of the Roman numerals if we try to multiply MMMDCXXVII by DLXXVIII. This illustration should prove clearly that the system of numbers given to us by the Arabs is much easier to work with than the Roman system. For a comparison of the Arabic and Roman numerals, see the table on page 61.

THE ARABIC SYSTEM In the Arabic system, there are ten symbols, called "digits": 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The value of a digit in a number depends wholly upon its position in that figure. In order for us to read the numbers written in the Arabic system, it is necessary to separate them by commas into groups (called "periods") of three figures in this manner: 4,866,587. This number is read "four million, eight hundred sixty-six thousand, five hundred eighty-seven". Beginning at the right, we call the first period the units' period; the second period, the thousands' period; and so on, as indicated in the table below.

Periods:

5th (Trillions)	4th (Billions)			3d (Millions)			2d (Thousands)			1st (Units)		
6 ,	5	4	0 ,	3	5	7 ,	4	2	3 ,	5	3	5
Trillions	Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units

The number presented in the example above is read "six trillion, five hundred forty billion, three hundred fifty-seven million, four hundred twenty-three thousand, five hundred thirty-five. You will note that the unit of any period equals 1,000 units of the next lower period; thus, the unit of the third period equals one thousand thousands, or one million. The British read 1,000,000,000 (our billion) as "one thousand millions", reserving the term, one billion, for 1,000,000,000,000.

To read an integral number expressed in figures, we begin at the right and point off the figures into periods of three figures in this manner:

85602579439

Begin at the right and point off the figures into periods of three figures each, thus:

85,602,579,439

Now we begin at the left and read each period as if it stood alone, adding the name of the period. We read the above number as "eighty-five billion, six hundred two million, five hundred seventy-nine thousand, four hundred thirty-nine".

To write an integral number in figures, we begin at the left and write the hundreds, tens, and units of each period, inserting zeros in all vacant places, and inserting a comma between each period and the period following it in this manner: "three hundred billion, four hundred twenty-two million, seventy thousand, one hundred eight". We first write the billions, insert a comma after it, then the period of millions, insert a comma after it, and so on. The final result is:

300,422,070,108.

The use of the zero

Suppose we try to write seven thousand fifty in decimal form. We write 7 and 5, but we have to show that there are gaps in the hundreds' and units' places; otherwise, the number would read as 75. At first, dots were used to show where there were gaps, and the number was written 7.5. Then someone thought of putting circles in the gaps, and the familiar form, 7050, was achieved! The zero merely shows that there is a gap—no units and no hundreds.

Notice that zeros before a number make no difference in the number. The expression, 0007050, still equals seven thousand fifty. We read the numbers only.

Obviously, there is no occasion, in the case of integers, to precede the numbers with zeros in most cases. We may omit them without in any way changing the value of the expression, and the writing of them would only occasion a loss of time and energy. Furthermore, the writing of these zeros would tend to make the column of figures somewhat more confusing. In a column of figures, then, unless all of the

numbers are composed of the same number of digits, we shall expect the left-hand edge to present a ragged appearance. By lining up the figures from the right-hand edge, counting place value as shown in the table on page 47, you need have no concern with these missing zeros. The spaces at the left are themselves equivalent to the zeros which you might have written there.

A zero after a number is, of course, a different thing because it moves the number one place to the left (which is equivalent to multiplying the number by ten). Then 70500 equals seventy thousand, five hundred, and 705000 equals seven hundred five thousand. If, however, we mark the units' figure in some way, then the zeros make no difference at either end. We could underline the units' figure, or put a cross over it, or write it in a box. It is a usual practice to put a dot (decimal point) after the units' figure to show which it is. Thus, 00093.000 = 93 because 3 is the units' figure, and the 9 to the left of it stands for 90. In British books, you will find the decimal point placed somewhat higher, as: 7·5.

DECIMALS

We have pointed out in the case of integers that a zero placed before a digit does not alter its value, but a zero placed after a digit does alter its value. In decimals, the zero placed before a digit does change its value, but a zero placed after a digit does not alter its value. Thus, 0.4 and 0.4000 are equal in value, but 0.4 and 0.0004 are quite different in value.

We read a decimal exactly as if it were a whole number, and then add the fractional name of the lowest place.

Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths	Hundred-millionths	Billionths
0	.	7	4	0,	0	3	5,	8	1	2

Read this number as follows: "seven hundred forty million, thirty-five thousand, eight hundred twelve *millionths*".

In the event that there is a whole number and a decimal, the point separates the whole number from the decimal, as 3.14, which is read "three and fourteen hundredths". The decimal point is read *and.* Numbers which are composed of a whole number and a decimal are called mixed decimals.

A warning about the decimal point: There is a story of a man who was poisoned by a decimal point. His physician wrote, or meant to write, .5 gr. on a prescription. The druggist who filled the prescription read 5 gr. and therefore inserted a fatal dose of a certain ingredient into the medicine. To avoid such tragedies, it is now common practice to write 0.5 gr. when there is any possibility of the decimal point being overlooked.

Monetary notation

In writing dollars and cents, we place the decimal point between the dollars and cents; thus, fifteen dollars and forty-five cents is written \$15.45. The numbers to the left of the decimal point are read as dollars, the numbers to the first two places to the right of the point are read as cents, and the number in the third place as mills. In a case where the number of cents is less than ten, we write a zero in the tenths' place at the right of the decimal point; thus, two dollars, ten cents, is written \$2.10; five dollars, eight cents, is written \$5.08.

TEST YOUR KNOWLEDGE OF NUMBER SYSTEMS WITH THESE EXERCISES

Express in the Arabic system of notation:

1 VIII
2 XIV
3 XXXV
4 LXXV
5 XCIV
6 DCCC
7 MCMXX
8 $\overline{\text{V}}$
9 MDCCCXCVI
10 MDLXXXIX

Read the following:

11 0.10
12 0.65
13 0.483
14 0.1425
15 1.812
16 3.1416
17 2.001
18 10.456
19 1001.0001
20 100,000.7643

Write the following in decimal form:

21 $\frac{5}{10}$
22 $\frac{7}{100}$
23 $\frac{44}{1000000}$
24 $\frac{1325}{100000}$
25 $\frac{3258}{100}$
26 Eight hundredths
27 Forty-five ten-thousandths
28 Ten thousand and twenty-five tenths
29 Three hundred hundredths
30 Seven hundred twenty-five ten-thousandths

The Measuring Rod

MANY times, people are apt to underestimate the importance of a knowledge of elementary arithmetic. Since skill in any other field of mathematics depends upon accuracy in the fundamentals, it is unwise for a student to attempt a study of the higher branches without having made sure of his proficiency in the fundamentals. The accompanying "test" is designed to aid the reader in determining his own accuracy and speed. If you have difficulty in completing this exercise in less than an hour, further study of the fundamentals would seem to be indicated. Each group of examples is followed by page references, to enable you to check up on the points in which you find yourself "stumped". The answers to these exercises will be printed in the next issue of PRACTICAL MATHEMATICS.

INTEGERS

ADDITION

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
8	6	3	709	193	1072	422	9986	55288	896
5	3	9	457	648	758	88	4982	58	6781
9	7	4	686	352	503	849	729	8465	228
7	8	6	496	962	1369	798	5034	4460	4004
9	2	4	893	467	937	629	370	907	4569
8	1	1	252	967	2550	761	5108	46529	819
7	9	7	?	?	?	?	?	?	?
6	4	5							
?	?	?							

(If you find any difficulty in arriving at the correct answers to the addition exercises, re-read pages 3 to 6. Check your answers by one of the methods suggested on pages 6 to 8.)

SUBTRACTION

11	**12**	**13**	**14**	**15**	**16**	**17**	**18**
3214	4916	4956	9689	9769	50000	6005	5321
− 433	− 119	−3749	−3997	−6692	− 9009	−1888	−4997
?	?	?	?	?	?	?	?

(If you find any difficulty in arriving at the correct answers to the subtraction exercises, re-read pages 9 to 11. Check your answers by one of the methods suggested on page 12.)

MULTIPLICATION

19	**20**	**21**	**22**	**23**	**24**
1728	4689	9247	5376	2163	6273
× 78	× 69	× 786	× 947	× 975	× 769
?	?	?	?	?	?

(If you find any difficulty in arriving at the correct answers to the multiplication exercises, re-read pages 12 to 14. Check your answers by one of the methods suggested on page 15.)

DIVISION

25 29)75920 **26** 84)89034 **27** 38)81627 **28** 897)68976 **29** 117)45144

(If you find any difficulty in arriving at the correct answers to the division exercises, re-read pages 18 to 21. Check yours answers by one of the methods suggested on pages 22 or 23.)

FRACTIONS AND MIXED NUMBERS

ADDITION

30	31	32	33	34	35	36	37	38
$\frac{3}{4}$	$\frac{2}{3}$	$\frac{11}{16}$	$48\frac{3}{4}$	$60\frac{5}{9}$	$32\frac{1}{8}$	$13\frac{17}{24}$	$24\frac{1}{6}$	$64\frac{5}{8}$
$\frac{2}{3}$	$\frac{7}{8}$	$\frac{2}{3}$	$47\frac{2}{3}$	$38\frac{5}{6}$	$15\frac{3}{4}$	$16\frac{1}{2}$	$34\frac{8}{9}$	$36\frac{3}{16}$
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$	$62\frac{5}{8}$	$27\frac{1}{3}$	$22\frac{1}{24}$	$72\frac{3}{4}$	$63\frac{2}{3}$	$29\frac{5}{24}$
$\frac{1}{3}$	$\frac{1}{8}$	$\frac{5}{16}$	$29\frac{11}{12}$	$42\frac{1}{3}$	$19\frac{1}{2}$	$27\frac{1}{8}$	$18\frac{7}{9}$	$32\frac{7}{8}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{8}$	$72\frac{1}{2}$	$24\frac{1}{2}$	$28\frac{2}{3}$	$52\frac{3}{32}$	$16\frac{5}{12}$	$19\frac{5}{12}$
?	?	?	$25\frac{3}{4}$	$38\frac{2}{3}$	$18\frac{15}{16}$	$24\frac{1}{16}$	$52\frac{2}{3}$	$72\frac{3}{4}$
			?	?	?	?	?	?

(If you find any difficulty in arriving at the correct answers when adding fractions and mixed numbers, re-read pages 31 to 33 and page 35.)

SUBTRACTION

39 $\frac{11}{12}-\frac{3}{12}=?$ **42** $\frac{3}{4}-\frac{7}{12}=?$ **45** $6\frac{1}{2}-4\frac{1}{4}=?$ **48** $48\frac{5}{8}-36\frac{3}{4}=?$

40 $\frac{7}{8}-\frac{5}{16}=?$ **43** $\frac{2}{3}-\frac{1}{6}=?$ **46** $16\frac{3}{8}-9\frac{1}{5}=?$ **49** $96\frac{1}{2}-79\frac{7}{8}=?$

41 $\frac{1}{2}-\frac{1}{3}=?$ **44** $\frac{5}{6}-\frac{1}{3}=?$ **47** $36\frac{1}{2}-23\frac{3}{4}=?$ **50** $125\frac{1}{8}-108\frac{3}{4}=?$

(If you find any difficulty in arriving at the correct answers when subtracting fractions and mixed numbers, re-read pages 31 to 33 and page 36.)

MULTIPLICATION

51 $24\times\frac{11}{16}=?$ **53** $\frac{2}{5}\times\frac{3}{4}\times\frac{1}{6}=?$ **55** $1\frac{1}{2}\times\frac{5}{6}=?$

52 $18\times\frac{15}{32}=?$ **54** $33\frac{1}{3}\times12\frac{1}{2}=?$ **56** $1\frac{1}{4}\times1\frac{2}{5}=?$

(If you find any difficulty in arriving at the correct answers when multiplying fractions and mixed numbers, re-read pages 27 to 28 and page 37.)

DIVISION

57 $4\div\frac{1}{2}=?$ **59** $3\frac{1}{3}\div\frac{5}{6}=?$ **61** $6\frac{1}{4}\div1\frac{1}{4}=?$

58 $1\frac{1}{2}\div\frac{3}{4}=?$ **60** $2\frac{1}{4}\div\frac{3}{8}=?$ **62** $7\frac{3}{8}\div5\frac{1}{6}=?$

(If you find any difficulty in arriving at the correct answers when dividing fractions and mixed numbers, re-read pages 29 to 30 and page 37.)

Odd Problems For Off Hours

UNDER this caption, we shall present in each issue of PRACTICAL MATHEMATICS a group of problems somewhat related to the subject-matter of the course, but involving "trick" relationships. We offer these as "brain-testers" or recreation to serve as a dessert for the more solid matter contained in the issue. You can have a lot of fun trying them out on yourself and on your friends. Incidentally, the solution of these problems will help you to strengthen your knowledge of some of the mathematical principles you have been studying.—*Editor.*

An Easy One for a Starter!

1 How can you subtract 45 from 45 and get 45 as an answer?

The Sailor in the Canteen

2 A canteen was located in a square room, with a doorway in each wall. A sailor paid one dollar to get in, spent at the canteen half of what he had left, and paid one dollar to get out. He went in and out of each of the four doorways in this manner and finally came out broke. How much money did he have at the beginning?

This One Solves Itself—on Paper

3 Two defense workers entered an oyster bar in a southern city. Parks wagered that he could eat more oysters than Davis. Parks ate ninety-nine, but Davis ate one hundred and won. How many did both eat? (When you try this orally on your friends, it's an even bet that they can't get the right answer!)

Profit and Loss

4 A man sold a second-hand automobile for \$270, and later bought it back for \$240. Without spending any money on repairs, he sold it again for \$300. How much money did he make as a result of these transactions?

Not So Easy as It Looks

5 If three cats can catch three rats in three minutes, how many cats can catch one hundred rats in one hundred minutes?

Taking Stock of the Cattle

6 A ranchman, when asked how many head of cattle he owned, replied that he was uncertain, but that he had ascertained that, when he counted them by twos, by threes, by fours, by fives, or by sixes, there was always one steer left over, but that, when he counted them by sevens, the grouping came out even. What is the smallest number which could possibly indicate the number of cattle on his ranch?

Unlucky for the Prisoners

7 Three American soldiers wished to cross a stream with three Japanese prisoners. Their canoe would hold only two occupants at a time. All of the soldiers, but only one of the prisoners, can paddle the canoe. For strategic reasons, it is obvious that

the number of prisoners on either side of the stream (including the occupants of the canoe) must at no time outnumber the soldiers on that side. How may the soldiers arrange the trips across the stream so that they may at all times be in command of the situation?

Get the English on This One!

8 How long will it take a squirrel to carry six ears of corn into his hollow if he takes in three ears at a time?

Horse-meat Avoided

9 A Kentucky colonel left a will bequeathing his racehorses to his three sons in the following manner: Robert, the eldest, was to receive one-half; Arthur was to have one-third; and Frank was to get one-ninth of the total number of horses. At the time of his death, there were only seventeen horses in the stable. Since seventeen can be divided neither by two, nor by three, nor by nine, the brothers were in considerable perplexity as to how the division could be performed equitably. A clever lawyer hit upon a scheme whereby the colonel's intentions could be carried out to the satisfaction of all concerned. How did he manage it?

Short but Dubious

10 What is the third and a half of a third of a half of ten?

A Grade-A Problem

11 Three members of a scouting party came upon a farmer's wife carrying her eggs to market, and tried to purchase her entire stock. As she had orders for four dozen eggs, she refused to sell that many. However, she sold the first scout half the number of eggs she had, plus half an egg. Then she sold to the second scout half of the eggs that remained, and half an egg. The third had to be contented with half of the remainder, and another half-egg. This left her just the four dozen which she needed to fill her orders. How many eggs did she have in her basket when the scouts met her?

Throwing Good Money after Bad

12 A man purchased five dollars' worth of groceries and tendered the clerk a fifty-dollar bill in payment. The grocer, not being able to make the change, went to a neighboring store to convert the large bill into smaller ones. He then returned and gave the customer the groceries and nine five-dollar bills. After the purchaser had left, the other storekeeper discovered that the fifty-dollar bill was a counterfeit and demanded that the grocer make good his loss. How much was the grocer out as a result of the entire transaction?

The Marines' Dessert

13 Five hungry marines entered a U.S.O. club and found that the only food remaining was a large pie, already cut for serving. The first ordered $\frac{3}{8}$ of the pie, the second $\frac{1}{4}$, the third $\frac{1}{8}$, and the fourth $\frac{1}{16}$. How much was left for the fifth marine?

Two More for Good Measure

14 These will test your ability to manipulate digits: (a) Use four nines to express one hundred. (b) Multiply four by three digits to make exactly five.

Strange Ways With Numbers

Mind Reading

1 Ask a friend to select any number less than ten, double it, add nine, divide by two, then subtract the original number. At this point, you tell him the answer to the problem which he has performed in his head. The answer, regardless of what number less than ten he selected, will always be four and one-half.

The Five Digits

2 Did you know that there are several five-digit numbers in which the first two digits when multiplied by the last three digits will give as a product a five-digit number in which the same digits appear, though in different order? Let us select one of these magic five-digit numbers and try it out. Suppose we take 42,678. According to the rule, we multiply 678 by 42:

$$\begin{array}{r} 678 \\ 42 \\ \hline 1356 \\ 2712 \\ \hline 28476 \end{array}$$

The solution is 28,476, which as you can see is 42,678 with the digits rearranged. Other numbers that will work in the same way are 75,231; 87,435; 57,834; 24,651.

The Mystic One

3 Among the many interesting mathematical curiosities, there is one which is perhaps the strangest of them all, the mystic number 1. At first glance, you may think that there is some trick to the following calculations, but upon closer examination you will see that each calculation is mathematically accurate.

$$\begin{array}{c} 2+1\times9=11 \\ 3+12\times9=111 \\ 4+123\times9=1111 \\ 5+1234\times9=11111 \\ 6+12345\times9=111111 \\ 7+123456\times9=1111111 \\ 8+1234567\times9=11111111 \\ 9+12345678\times9=111111111 \\ 10+123456789\times9=1111111111 \end{array}$$

Note also this further peculiarity: in each instance, the number of digits in the answer is the same as the number which is added to the multiple of 9.

The Magic Square

4 Did you know that you can arrange the numbers from 1 through 9 into a "magic square" in such a way that all of the columns and rows, and each of the diagonals will add up to 15? Here is how it is done:

4	9	2
3	5	7
8	1	6

No matter in what direction you add, whether it be each column, row, or diagonal, the sum of the figures will always equal fifteen.

It is also possible to construct a "magic square" using the numbers from 1 through 16. There are 880

different ways in which this "magic square" may be constructed. As in the case of the magic square using nine numbers, no matter what way you add the figures the sum of the numbers will always remain the same. In this case, the sum is always 34.

12	13	8	1
7	2	11	14
9	16	5	4
6	3	10	15

As yet, no one has figured out how many ways a magic square using all of the numbers between 1 and 25 may be constructed, but the number probably runs into the thousands. Here is a typical "magic square", in which twenty-five numbers are used and in which each row, column, and diagonal adds up to 65.

15	8	1	24	17
16	14	7	5	23
22	20	13	6	4
3	21	19	12	10
9	2	25	18	11

In the event that the inner square of nine figures is itself a "magic square", the number of magic squares that can be developed from these figures is 174,240.

Rotating Digits

5 Of many unusual numbers in mathematics, none has so many strange peculiarities as the magic number, 76923. When this number is multiplied by 1, 2, 3, 4, 5, and so on up to 12, the answers will consist of the same digits arranged in different orders, and these may be grouped in such a way that they appear in the same sequence of digits when they are read from left to right or from top to bottom. You will note this additional peculiarity; in each of the following groups, the digits in the solutions move up one place in each answer. In the answer to the first problem in the illustration, you will note that the digit, 3, is last in the line of numbers. In the answer to the second problem, the 3 has moved up one space, and the zero which was in first place has moved to the end of the line of numbers, while each of the other digits has moved up one space. In each successive solution, the digit, 3, moves up one space, until in the final solution the 3 has moved into first place. You will note also that all of the digits change position according to this regular form in each successive step.

In the first illustration let us multiply 76923 by 1, 3, 4, 9, 10, and 12, and arrange the solutions in their proper order.

$$1 \times 76923 = 076923$$
$$10 \times 76923 = 769230$$
$$9 \times 76923 = 692307$$
$$12 \times 76923 = 923076$$
$$3 \times 76923 = 230769$$
$$4 \times 76923 = 307692$$

Now, let us use the remaining numbers between 1 and 12, treating the magic number as we have done above. This time, we arrive at a different solution; that is, that the answers are made up of a different combination of digits. The same digits appear in all solutions, however, and in each successive solution the digits shift places as they did in the first illustration.

$$2 \times 76923 = 153846$$
$$7 \times 76923 = 538461$$
$$5 \times 76923 = 384615$$
$$11 \times 76923 = 846153$$
$$6 \times 76923 = 461538$$
$$8 \times 76923 = 615384$$

Tables and Formulas

TABLE I

SIMPLE ADDITION FACTS

$\begin{array}{r}0\\0\\\hline 0\end{array}$	$\begin{array}{r}1\\0\\\hline 1\end{array}$	$\begin{array}{r}0\\1\\\hline 1\end{array}$	$\begin{array}{r}2\\0\\\hline 2\end{array}$	$\begin{array}{r}0\\2\\\hline 2\end{array}$	$\begin{array}{r}3\\0\\\hline 3\end{array}$	$\begin{array}{r}0\\3\\\hline 3\end{array}$	$\begin{array}{r}4\\0\\\hline 4\end{array}$	$\begin{array}{r}0\\4\\\hline 4\end{array}$	$\begin{array}{r}5\\0\\\hline 5\end{array}$	$\begin{array}{r}0\\5\\\hline 5\end{array}$	$\begin{array}{r}6\\0\\\hline 6\end{array}$	$\begin{array}{r}0\\6\\\hline 6\end{array}$	$\begin{array}{r}7\\0\\\hline 7\end{array}$	$\begin{array}{r}0\\7\\\hline 7\end{array}$	$\begin{array}{r}8\\0\\\hline 8\end{array}$	$\begin{array}{r}0\\8\\\hline 8\end{array}$	$\begin{array}{r}9\\0\\\hline 9\end{array}$	$\begin{array}{r}0\\9\\\hline 9\end{array}$
		$\begin{array}{r}1\\1\\\hline 2\end{array}$	$\begin{array}{r}2\\1\\\hline 3\end{array}$	$\begin{array}{r}1\\2\\\hline 3\end{array}$	$\begin{array}{r}3\\1\\\hline 4\end{array}$	$\begin{array}{r}1\\3\\\hline 4\end{array}$	$\begin{array}{r}4\\1\\\hline 5\end{array}$	$\begin{array}{r}1\\4\\\hline 5\end{array}$	$\begin{array}{r}5\\1\\\hline 6\end{array}$	$\begin{array}{r}1\\5\\\hline 6\end{array}$	$\begin{array}{r}6\\1\\\hline 7\end{array}$	$\begin{array}{r}1\\6\\\hline 7\end{array}$	$\begin{array}{r}7\\1\\\hline 8\end{array}$	$\begin{array}{r}1\\7\\\hline 8\end{array}$	$\begin{array}{r}8\\1\\\hline 9\end{array}$	$\begin{array}{r}1\\8\\\hline 9\end{array}$	$\begin{array}{r}9\\1\\\hline 10\end{array}$	$\begin{array}{r}1\\9\\\hline 10\end{array}$
				$\begin{array}{r}2\\2\\\hline 4\end{array}$	$\begin{array}{r}3\\2\\\hline 5\end{array}$	$\begin{array}{r}2\\3\\\hline 5\end{array}$	$\begin{array}{r}4\\2\\\hline 6\end{array}$	$\begin{array}{r}2\\4\\\hline 6\end{array}$	$\begin{array}{r}5\\2\\\hline 7\end{array}$	$\begin{array}{r}2\\5\\\hline 7\end{array}$	$\begin{array}{r}6\\2\\\hline 8\end{array}$	$\begin{array}{r}2\\6\\\hline 8\end{array}$	$\begin{array}{r}7\\2\\\hline 9\end{array}$	$\begin{array}{r}2\\7\\\hline 9\end{array}$	$\begin{array}{r}8\\2\\\hline 10\end{array}$	$\begin{array}{r}2\\8\\\hline 10\end{array}$	$\begin{array}{r}9\\2\\\hline 11\end{array}$	$\begin{array}{r}2\\9\\\hline 11\end{array}$
						$\begin{array}{r}3\\3\\\hline 6\end{array}$	$\begin{array}{r}4\\3\\\hline 7\end{array}$	$\begin{array}{r}3\\4\\\hline 7\end{array}$	$\begin{array}{r}5\\3\\\hline 8\end{array}$	$\begin{array}{r}3\\5\\\hline 8\end{array}$	$\begin{array}{r}6\\3\\\hline 9\end{array}$	$\begin{array}{r}3\\6\\\hline 9\end{array}$	$\begin{array}{r}7\\3\\\hline 10\end{array}$	$\begin{array}{r}3\\7\\\hline 10\end{array}$	$\begin{array}{r}8\\3\\\hline 11\end{array}$	$\begin{array}{r}3\\8\\\hline 11\end{array}$	$\begin{array}{r}9\\3\\\hline 12\end{array}$	$\begin{array}{r}3\\9\\\hline 12\end{array}$
								$\begin{array}{r}4\\4\\\hline 8\end{array}$	$\begin{array}{r}5\\4\\\hline 9\end{array}$	$\begin{array}{r}4\\5\\\hline 9\end{array}$	$\begin{array}{r}6\\4\\\hline 10\end{array}$	$\begin{array}{r}4\\6\\\hline 10\end{array}$	$\begin{array}{r}7\\4\\\hline 11\end{array}$	$\begin{array}{r}4\\7\\\hline 11\end{array}$	$\begin{array}{r}8\\4\\\hline 12\end{array}$	$\begin{array}{r}4\\8\\\hline 12\end{array}$	$\begin{array}{r}9\\4\\\hline 13\end{array}$	$\begin{array}{r}4\\9\\\hline 13\end{array}$
										$\begin{array}{r}5\\5\\\hline 10\end{array}$	$\begin{array}{r}6\\5\\\hline 11\end{array}$	$\begin{array}{r}5\\6\\\hline 11\end{array}$	$\begin{array}{r}7\\5\\\hline 12\end{array}$	$\begin{array}{r}5\\7\\\hline 12\end{array}$	$\begin{array}{r}8\\5\\\hline 13\end{array}$	$\begin{array}{r}5\\8\\\hline 13\end{array}$	$\begin{array}{r}9\\5\\\hline 14\end{array}$	$\begin{array}{r}5\\9\\\hline 14\end{array}$
												$\begin{array}{r}6\\6\\\hline 12\end{array}$	$\begin{array}{r}7\\6\\\hline 13\end{array}$	$\begin{array}{r}6\\7\\\hline 13\end{array}$	$\begin{array}{r}8\\6\\\hline 14\end{array}$	$\begin{array}{r}6\\8\\\hline 14\end{array}$	$\begin{array}{r}9\\6\\\hline 15\end{array}$	$\begin{array}{r}6\\9\\\hline 15\end{array}$
														$\begin{array}{r}7\\7\\\hline 14\end{array}$	$\begin{array}{r}8\\7\\\hline 15\end{array}$	$\begin{array}{r}7\\8\\\hline 15\end{array}$	$\begin{array}{r}9\\7\\\hline 16\end{array}$	$\begin{array}{r}7\\9\\\hline 16\end{array}$
																$\begin{array}{r}8\\8\\\hline 16\end{array}$	$\begin{array}{r}9\\8\\\hline 17\end{array}$	$\begin{array}{r}8\\9\\\hline 17\end{array}$
																		$\begin{array}{r}9\\9\\\hline 18\end{array}$

TABLE II

SHORT-CUTS IN MULTIPLYING

To Multiply By	Annex	Then Divide By
5	One zero	2
10	One zero	–
25	Two zeros	4
50	Two zeros	2
75	Two zeros; multiply by 3	4
100	Two zeros	–
125	Three zeros	8
250	Three zeros	4
500	Three zeros	2
750	Three zeros; multiply by 3	4
1000	Three zeros	–

Explanation: Since 25 is $\frac{1}{4}$ of 100, multiply by 100 (which can be done simply by annexing two zeros) and divide by 4, as: $186 \times 25 = 18600 \div 4 = 4650$, etc.

In the case of 75 and 750, it is sometimes easier to multiply by 3 before annexing the zeros, as: $186 \times 75 = (186 \times 3 = 558)\ 55800 \div 4 = 13950$, than to proceed as in the case of 25 and multiply by 3 as the last step.

TABLE III

SHORT-CUTS IN DIVISION

To Divide By	Move Decimal Point	Then
5	ONE place to left	Multiply by 2
10	ONE place to left	——
20	ONE place to left (or) TWO places to left	Divide by 2 Multiply by 5
25	TWO places to left	Multiply by 4
50	TWO places to left	Multiply by 2
75	TWO places to left	Multiply by 4, divide by 3
100	TWO places to left	——
125	THREE places to left	Multiply by 8
250	THREE places to left	Multiply by 4
500	THREE places to left	Multiply by 2
750	THREE places to left	Multiply by 4, divide by 3
1000	THREE places to left	——

TABLE IV

DETERMINING THE DIVISIBILITY OF NUMBERS

Before attempting to divide a large number, it is helpful to make a quick survey to determine whether or not the number is evenly divisible by a given factor. It will pay you to memorize these rules and to practice them sufficiently so that you may apply them readily:

Divisible By	When Number Ends In	Or
2	Any even number (or) Zero	
3		When SUM of digits can be divided by 3
4	Two zeros (or) Multiple of 20	When LAST TWO digits can be divided by 4
5	Zero or 5	
6		When number is even and SUM of digits can by divided by 3
7		When SUM of odd-place digits MINUS SUM of even-place digits can be divided by 7*
8	Three zeros (or) Multiple of 200	When LAST THREE digits can be divided by 8
9		When SUM of ALL digits can be divided by 9
10	Zero	
11		When SUM of odd-place digits MINUS SUM of even-place digits can be divided by 11
12	Multiple of 300	When SUM of digits can be divided by 3 and EACH of LAST TWO digits can be divided by 4.

* Usually true for arithmetic subtraction.

TABLE V

FACTORS OF POSITIVE INTEGERS

(Since the number, 1, is a factor of every number it is omitted in the listings. Prime numbers, those divisible only by themselves and by 1, are indicated by a star.)

		51	3,17	**101**	*	**151**	*	**201**	3,67
2	*	**52**	2,2,13	**102**	2,3,17	**152**	2,2,2,19	**202**	2,101
3	*	**53**	*	**103**	*	**153**	3,3,17	**203**	7,29
4	2,2	**54**	2,3,3,3	**104**	2,2,2,13	**154**	2,7,11	**204**	2,2,3,17
5	*	**55**	5,11	**105**	3,5,7	**155**	5,31	**205**	5,41
6	2,3	**56**	2,2,2,7	**106**	2,53	**156**	2,2,3,13	**206**	2,103
7	*	**57**	3,19	**107**	*	**157**	*	**207**	3,3,23
8	2,2,2	**58**	2,29	**108**	2,2,3,3,3	**158**	2,79	**208**	2,2,2,2,13
9	3,3	**59**	*	**109**	*	**159**	3,53	**209**	11,19
10	2,5	**60**	2,2,3,5	**110**	2,5,11	**160**	2,2,2,2,2,5	**210**	2,3,5,7
11	*	**61**	*	**111**	3,37	**161**	7,23	**211**	*
12	2,2,3	**62**	2,31	**112**	2,2,2,2,7	**162**	2,3,3,3,3	**212**	2,2,53
13	*	**63**	3,3,7	**113**	*	**163**	*	**213**	3,71
14	2,7	**64**	2,2,2,2,2,2	**114**	2,3,19	**164**	2,2,41	**214**	2,107
15	3,5	**65**	5,13	**115**	5,23	**165**	3,5,11	**215**	5,43
16	2,2,2,2	**66**	2,3,11	**116**	2,2,29	**166**	2,83	**216**	2,2,2,3,3,3
17	*	**67**	*	**117**	3,3,13	**167**	*	**217**	7,31
18	2,3,3	**68**	2,2,17	**118**	2,59	**168**	2,2,2,3,7	**218**	2,109
19	*	**69**	3,23	**119**	7,17	**169**	13,13	**219**	3,73
20	2,2,5	**70**	2,5,7	**120**	2,2,2,3,5	**170**	2,5,17	**220**	2,2,5,11
21	3,7	**71**	*	**121**	11,11	**171**	3,3,19	**221**	13,17
22	2,11	**72**	2,2,2,3,3	**122**	2,61	**172**	2,2,43	**222**	2,3,37
23	*	**73**	*	**123**	3,41	**173**	*	**223**	*
24	2,2,2,3	**74**	2,37	**124**	2,2,31	**174**	2,3,29	**224**	2,2,2,2,2,7
25	5,5	**75**	3,5,5	**125**	5,5,5	**175**	5,5,7	**225**	3,3,5,5
26	2,13	**76**	2,2,19	**126**	2,3,3,7	**176**	2,2,2,2,11	**226**	2,113
27	3,3,3	**77**	7,11	**127**	*	**177**	3,59	**227**	*
28	2,2,7	**78**	2,3,13	**128**	2,2,2,2,2,2,2	**178**	2,89	**228**	2,2,3,19
29	*	**79**	*	**129**	3,43	**179**	*	**229**	*
30	2,3,5	**80**	2,2,2,2,5	**130**	2,5,13	**180**	2,2,3,3,5	**230**	2,5,23
31	*	**81**	3,3,3,3	**131**	*	**181**	*	**231**	3,7,11
32	2,2,2,2,2	**82**	2,41	**132**	2,2,3,11	**182**	2,7,13	**232**	2,2,2,29
33	3,11	**83**	*	**133**	7,19	**183**	3,61	**233**	*
34	2,17	**84**	2,2,3,7	**134**	2,67	**184**	2,2,2,23	**234**	2,3,3,13
35	5,7	**85**	5,17	**135**	3,3,3,5	**185**	5,37	**235**	5,47
36	2,2,3,3	**86**	2,43	**136**	2,2,2,17	**186**	2,3,31	**236**	2,2,59
37	*	**87**	3,29	**137**	*	**187**	11,17	**237**	3,79
38	2,19	**88**	2,2,2,11	**138**	2,3,23	**188**	2,2,47	**238**	2,7,17
39	3,13	**89**	*	**139**	*	**189**	3,3,3,7	**239**	*
40	2,2,2,5	**90**	2,3,3,5	**140**	2,2,5,7	**190**	2,5,19	**240**	2,2,2,2,3,5
41	*	**91**	7,13	**141**	3,37	**191**	*	**241**	*
42	2,3,7	**92**	2,2,23	**142**	2,71	**192**	2,2,2,2,2,2,3	**242**	2,11,11
43	*	**93**	3,31	**143**	11,13	**193**	*	**243**	3,3,3,3,3
44	2,2,11	**94**	2,47	**144**	2,2,2,2,3,3	**194**	2,97	**244**	2,2,61
45	3,3,5	**95**	5,19	**145**	5,29	**195**	3,5,13	**245**	5,7,7
46	2,23	**96**	2,2,2,2,2,3	**146**	2,73	**196**	2,2,7,7	**246**	2,3,41
47	*	**97**	*	**147**	3,7,7	**197**	*	**247**	13,19
48	2,2,2,2,3	**98**	2,7,7	**148**	2,2,37	**198**	2,3,3,11	**248**	2,2,2,31
49	7,7	**99**	3,3,11	**149**	*	**199**	*	**249**	3,83
50	2,5,5	**100**	2,2,5,5	**150**	2,3,5,5	**200**	2,2,2,5,5	**250**	2,5,5,5

TABLE V (continued)

FACTORS OF POSITIVE INTEGERS

251	*	**301**	7,43	**351**	3,3,3,19	**401**	*	**451**	11,41
252	2,2,3,3,7	**302**	2,151	**352**	2,2,2,2,2,11	**402**	2,3,67	**452**	2,2,113
253	11,23	**303**	3,101	**353**	*	**403**	13,31	**453**	3,151
254	2,127	**304**	2,2,2,2,19	**354**	2,3,59	**404**	2,2,101	**454**	2,227
255	3,5,17	**305**	5,61	**355**	5,71	**405**	3,3,3,3,5	**455**	5,7,13
256	2,2,2,2,2,2,2,2	**306**	2,3,3,17	**356**	2,2,89	**406**	2,7,29	**456**	2,2,2,3,19
257	*	**307**	*	**357**	3,7,17	**407**	11,37	**457**	*
258	2,3,43	**308**	2,2,7,11	**358**	2,179	**408**	2,2,2,3,17	**458**	2,229
259	7,37	**309**	3,103	**359**	*	**409**	*	**459**	3,3,3,17
260	2,2,5,13	**310**	2,5,31	**360**	2,2,2,3,3,5	**410**	2,5,41	**460**	2,2,5,23
261	3,3,29	**311**	*	**361**	19,19	**411**	3,137	**461**	*
262	2,131	**312**	2,2,2,3,13	**362**	2,181	**412**	2,2,103	**462**	2,3,7,11
263	*	**313**	*	**363**	3,11,11	**413**	7,59	**463**	*
264	2,2,2,3,11	**314**	2,157	**364**	2,2,7,13	**414**	2,3,3,23	**464**	2,2,2,2,29
265	5,53	**315**	3,3,5,7	**365**	5,73	**415**	5,83	**465**	3,5,31
266	2,7,19	**316**	2,2,79	**366**	2,3,61	**416**	2,2,2,2,2,13	**466**	2,233
267	3,89	**317**	*	**367**	*	**417**	3,139	**467**	*
268	2,2,67	**318**	2,3,53	**368**	2,2,2,2,23	**418**	2,11,19	**468**	2,2,3,3,13
269	*	**319**	11,29	**369**	3,3,41	**419**	*	**469**	7,67
270	2,3,3,3,5	**320**	2,2,2,2,2,2,5	**370**	2,5,37	**420**	2,2,3,5,7	**470**	2,5,47
271	*	**321**	3,107	**371**	7,53	**421**	*	**471**	3,157
272	2,2,2,2,17	**322**	2,7,23	**372**	2,2,3,31	**422**	2,211	**472**	2,2,2,59
273	3,7,13	**323**	17,19	**373**	*	**423**	3,3,47	**473**	11,43
274	2,137	**324**	2,2,3,3,3,3	**374**	2,11,17	**424**	2,2,2,53	**474**	2,3,79
275	5,5,11	**325**	5,5,13	**375**	3,5,5,5	**425**	5,5,17	**475**	5,5,19
276	2,2,3,23	**326**	2,163	**376**	2,2,2,47	**426**	2,3,71	**476**	2,2,7,17
277	*	**327**	3,109	**377**	13,29	**427**	7,61	**477**	3,3,53
278	2,139	**328**	2,2,2,41	**378**	2,3,3,3,7	**428**	2,2,107	**478**	2,239
279	3,3,31	**329**	7,47	**379**	*	**429**	3,11,13	**479**	*
280	2,2,2,5,7	**330**	2,3,5,11	**380**	2,2,5,19	**430**	2,5,43	**480**	2,2,2,2,2,3,5
281	*	**331**	*	**381**	3,127	**431**	*	**481**	13,37
282	2,3,47	**332**	2,2,83	**382**	2,191	**432**	2,2,2,2,3,3,3	**482**	2,241
283	*	**333**	3,3,37	**383**	*	**433**	*	**483**	3,7,23
284	2,2,71	**334**	2,167	**384**	2,2,2,2,2,2,2,3	**434**	2,7,31	**484**	2,2,11,11
285	3,5,19	**335**	5,67	**385**	5,7,11	**435**	3,5,29	**485**	5,97
286	2,11,13	**336**	2,2,2,2,3,7	**386**	2,193	**436**	2,2,109	**486**	2,3,3,3,3,3
287	7,41	**337**	*	**387**	3,3,43	**437**	19,23	**487**	*
288	2,2,2,2,2,3,3	**338**	2,13,13	**388**	2,2,97	**438**	2,3,73	**488**	2,2,2,61
289	17,17	**339**	3,113	**389**	*	**439**	*	**489**	3,163
290	2,5,29	**340**	2,2,5,17	**390**	2,3,5,13	**440**	2,2,2,5,11	**490**	2,5,7,7
291	3,97	**341**	11,31	**391**	17,23	**441**	3,3,7,7	**491**	*
292	2,2,73	**342**	2,3,3,19	**392**	2,2,2,7,7	**442**	2,13,17	**492**	2,2,3,41
293	*	**343**	7,7,7	**393**	3,131	**443**	*	**493**	17,29
294	2,3,7,7	**344**	2,2,2,43	**394**	2,197	**444**	2,2,3,37	**494**	2,13,19
295	5,59	**345**	3,5,23	**395**	5,79	**445**	5,89	**495**	3,3,5,11
296	2,2,2,37	**346**	2,173	**396**	2,2,3,3,11	**446**	2,223	**496**	2,2,2,2,31
297	3,3,3,11	**347**	*	**397**	*	**447**	3,149	**497**	7,71
298	2,149	**348**	2,2,3,29	**398**	2,199	**448**	2,2,2,2,2,2,7	**498**	2,3,83
299	13,23	**349**	*	**399**	3,7,19	**449**	*	**499**	*
300	2,2,3,5,5	**350**	2,5,5,7	**400**	2,2,2,2,5,5	**450**	2,3,3,5,5	**500**	2,2,5,5,5

TABLE VI

ROMAN AND ARABIC NUMBER SYSTEMS COMPARED

Arabic	Word	Roman	Arabic	Word	Roman
1	one	I	30	thirty	XXX
2	two	II	40	forty	XL
3	three	III	50	fifty	L
4	four	IV	60	sixty	LX
5	five	V	70	seventy	LXX
6	six	VI	80	eighty	LXXX
7	seven	VII	90	ninety*	XC
8	eight	VIII	100	one hundred	C
9	nine	IX	101	one hundred one	CI
10	ten	X	110	one hundred ten	CX
11	eleven	XI	200	two hundred	CC
12	twelve	XII	300	three hundred	CCC
13	thirteen	XIII	400	four hundred*	CD
14	fourteen	XIV	500	five hundred	D
15	fifteen	XV	600	six hundred	DC
16	sixteen	XVI	700	seven hundred	DCC
17	seventeen	XVII	800	eight hundred	DCCC
18	eighteen	XVIII	900	nine hundred*	CM
19	nineteen	XIX	1000	one thousand	M
20	twenty	XX			
21	twenty-one	XXI			

* Usage varies

TABLE VII

PLACE VALUES OF NUMBERS

Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths	Millionths	Ten millionths	Hundred millionths	Billionths	
									1											one
								1	0											ten
							1	0	0											one hundred
						1,	0	0	0											one thousand
					1	0,	0	0	0											ten thousand
				1	0	0,	0	0	0											one hundred thousand
			1,	0	0	0,	0	0	0											one million
		1	0,	0	0	0,	0	0	0											ten million
	1	0	0,	0	0	0,	0	0	0											one hundred million
1,	0	0	0,	0	0	0,	0	0	0											one billion
										.										decimal point (read "and")
										.	1									one tenth
										.	0	1								one hundredth
										.	0	0	1							one thousandth
										.	0	0	0,	1						one ten thousandth
										.	0	0	0,	0	1					one hundred thousandth
										.	0	0	0,	0	0	1				one millionth
										.	0	0	0,	0	0	0,	1			one ten millionth
										.	0	0	0,	0	0	0,	0	1		one hundred millionth
										.	0	0	0,	0	0	0,	0	0	1	one billionth

TABLE VIII

COMMON WEIGHTS AND MEASURES

ENGLISH | METRIC

Linear Measure

12 inches (in.) = 1 foot (ft.)
36 in. = 3 ft. = 1 yard (yd.)
$16\frac{1}{2}$ ft. = $5\frac{1}{2}$ yd. = 1 rod (rd.)
5280 ft. = 1760 yd. = 320 rd. = 1 mile (mi.)

10 millimeters (mm.) = 1 centimeter (cm.)
10 cm. = 1 decimeter (dm.)
10 dm. = 1 meter (m.)
10 m. = 1 Dekameter (Dm.)
10 Dm. = 1 Hektometer (Hm.)
10 Hm. = 1 kilometer (Km.)

Square Measure

144 sq. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
$30\frac{1}{4}$ sq. yd. = 1 sq. rd.
160 sq. rd. = 1 acre
640 acres = 1 sq. mi.

100 sq. mm. = 1 sq. cm.
100 sq. cm. = 1 sq. dm.
100 sq. dm. = 1 sq. m.
100 sq. m. = 1 sq. Dm.
100 sq. Dm. = 1 sq. Hm.
100 sq. Hm. = 1 sq. Km.

Cubic Measure

1,728 cu. in. = 1 cu. ft.
46,656 cu. in. = 27 cu. ft. = 1 cu. yd.

1000 cu. mm. = 1 cu. cm.
1000 cu. cm. = 1 cu. dm.
1000 cu. dm. = 1 cu. m.

Liquid Measure

4 gills = 1 pint (pt.)
2 pt. = 1 quart (qt.)
4 qt. = 1 gallon (gal.)

10 centiliters (cl.) = 1 deciliter (dl.)
10 dl. = 1 liter (l.)
10 l. = 1 Dekaliter (Dl.)
10 Dl. = 1 Hektoliter (Hl.)
10 Hl. = 1 kiloliter (Kl.)

Dry Measure

2 cups = 1 pt.
2 pt. = 1 qt.
4 qt. = 1 gal.
8 qt. = 1 peck (pk.)
4 pk. = 1 bushel (bu.)

10 centiliters (cl.) = 1 deciliter (dl.)
10 dl. = 1 liter (l.)
10 l. = 1 Dekaliter (Dl.)
10 Dl. = 1 Hektoliter (Hl.)
10 Hl. = 1 kiloliter (Kl.)

Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.)
2000 lb. = 1 ton
2240 lb. = 1 long ton

10 centigrams (cg.) = 1 decigram (dg.)
10 dg. = 1 gram (g.)
10 g. = 1 Dekagram (Dg.)
10 Dg. = 1 Hectogram (Hg.)
10 Hg. = 1 kilogram (Kg.)
1000 kg. = 1 metric ton (T.)

Time Measure

60 seconds (sec.) = 1 minute (min.)
60 min. = 1 hour (hr.)
24 hr. = 1 day
7 days = 1 week
365 days = 52 weeks = 1 year (yr.)

Glossary of Mathematical Terms

addend: a number to be added, or one of several numbers to be added. Given 3 +2, 2 is the addend for 3; given 25 +436 +763, the numbers, 25, 436, and 763 are considered addends.

addition: the process of adding numbers together. In arithmetic, numbers are added by counting the whole number of units in the groups to be added. (See page 3.)

approximate: nearly correct, or sufficiently correct for the purpose of the problem.

Arabic numerals: the designation for the numbers commonly used: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. So called because these numbers were introduced into Europe (and from there to America) from Arabia. (See page 39.)

arithmetic: the study of numbers and the operations which may be performed with them (See *fundamental operations*) and the application of these results in solving problems arising in every-day living.

borrowing: an operation in subtraction. In such a problem as 43 − 27, we cannot subtract 7 from 3. We borrow 10 from the 40, combining it with the 3 to make 13. Then we subtract 7 from 13 and 20 from 30 to get the answer. (See page 10.)

canceling: striking out like factors which appear in both numerator and denominator of a fraction. (See page 29.)

carrying: the process of removing a quantity from one column to another. In addition or multiplication, whenever we get a figure which is expressed in more than one digit, (as 4 × 4 = 16), we set down the 6 in its own column and carry the 1 (representing 10) to the next column to the left. (See pages 4 and 12.)

checking: determining the accuracy of a computation by repeating the operation, or using another operation and comparing the results.

cipher: sometimes used to designate zero (0).

column: numbers arranged in a vertical row.

common fraction: a fraction whose numerator is less than its denominator $\left(\text{as } \frac{2}{3}, \frac{8}{15}\right)$.

common multiple: a number which is a multiple of each of two or more given numbers. (See *multiple.*)

consecutive: without a gap. The series, 2, 3, 4, 5, is consecutive; 2, 4, 5 is not.

correct: accurate, without error.

count: to name a set of consecutive numbers in order.

cross-multiplication: multiplying the numerator of one fraction by the denominator of another. (See page 32.)

decimal: the system of counting by tens. (*Decem* is the Latin word for *ten.*)

decimal fraction: Arabic numerals following a decimal point to indicate less than unity, as 0.25, which represents twenty-five one-hundredths.

decimal point: the period separating a whole number from a decimal fraction.

denominator: the number below a fraction bar. In the fraction, $\frac{3}{4}$, 4 is the denominator. (See page 25.)

difference: the amount found by subtracting one number from another.

digit: any one of the numbers from 1 to 9, place value being ignored. In the number, 34, the digits are 3 and 4. (*Digit* comes from the Latin, *digitus*, meaning *finger*. The primitive custom of counting on the fingers gave rise to this designation of simple numbers.)

dividend: a number to be divided by another number.

division: separating a number into equal parts. (See page 18.)

divisor: the quantity by which a number is to be divided.

duodecimal: the system of counting by twelves. (*Duodecim* is the Latin word for *twelve.*)

equality: a condition in which one number or set of factors represents the same amount as another number or set of factors.

equation: a statement that two quantities or sets of quantities are equal.

even number: any number that is divisible by 2 without a remainder.

excess: an amount greater than a specified amount.

factor: one of a group of numbers which, multiplied together, produce a given number. (See page 59.)

fractions: a form of representing the division of one number by another, in which the *dividend* is placed in the *numerator*, above a fraction-bar, and the *divisor* is placed in the *denominator*, under the bar. (See also *common fractions*, *proper fractions*, *improper fractions*, and the italicized words in the preceding definition.) A number less than unity. (See also *decimal fraction.*)

horizontal: extending across the page from left to right.

identity: a statement that values are always equal.

improper fractions: fractions whose numerators are larger than their denominators $\left(\text{as } \frac{4}{3}, \frac{12}{7}\right)$.

integer: a whole number.

invert: to turn over. In inverting a fraction, we reverse the positions of numerator and denominator $\left(\text{as } \frac{2}{3} \text{ inverted becomes } \frac{3}{2}\right)$.

least common multiple: the smallest number that is divisible without remainder by each of several given numbers.

lowestcommondenominator (abbreviated L.C.D.)**:** the smallest number divisible by all the denominators in a group of fractions. (See page 31.)

lowest terms: used in speaking of fractions, to indicate the fraction resulting when all common factors have been divided out of the numerator and denominator. (See page 27.)

long division: usually applied to any division in which the divisor contains more than one digit, because the steps in division must be taken separately. (See page 20.)

mathematics: the study of shape, arrangement, and quantity, pursued in a logical manner.

minus: decreased by. Used in subtraction as a reading for the sign of subtraction: $6-2$, six minus two. (The Latin word, *minus*, means *less*.)

minuend: a number from which another number is to be subtracted.

mixed number: a number composed of an integer and a fraction. The fraction may be either common or decimal. Both of these expressions are mixed numbers: $2\frac{1}{2}$, 3.75. Any *improper fraction* may be converted into a mixed number by dividing the numerator by the denominator $\left(\text{as } \frac{7}{2}=3\frac{1}{2}\right)$.

multiple: a quantity which is the product of a given number and another factor. Thus, 6 and 9 are both multiples of 3, since $3\times2=6$ and $3\times3=9$. (See page 17.)

multiplicand: a number which is to be multiplied by another number.

multiplier: a number by which another number is multiplied.

multiplication: the process of taking a number a given number of times; a short form of addition. (See page 12.)

number: the name for how many times a thing is to be taken; in arithmetic, practically synonymous with *integer*. (See page 37.)

numeral: the symbol used to express a number. (See page 39.)

numerator: the number above a fraction bar. In the fraction, $\frac{3}{4}$, 3 is the numerator. (See page 25).

odd number: a number that cannot be divided without remainder by 2.

parentheses: the symbols used to enclose a grouping of sums or products, usually employed in pairs: the beginning parenthesis, (, and the ending parenthesis,).

parenthesis: the expression enclosed in parentheses to be taken as one quantity, all operations within the parenthesis to be performed before the parenthesis is removed.

period: digits set off by a comma when a number is expressed in Arabic numerals. (See page 39.)

placement: the value which an Arabic numeral receives by virtue of its position in a line of figures. (See page 40.)

plus: added to. Used in addition as the reading for the sign of addition, to mean *and*. (The Latin word, *plus*, means *more*.)

point: See *decimal point*.

prime number: a number which can be divided only by itself and by 1 without leaving a remainder.

product: the result obtained when two or more numbers are multiplied together.

proper fraction: same as *common fraction*.

quotient: the number obtained by dividing one number by another.

reduction: dividing both numerator and denominator of a fraction in order that the fraction may be expressed in *lowest terms*. (See page 30.)

result: the answer to a computation or problem.

resultant: an answer obtained by performing a given operation.

Roman numerals: a system of expressing numbers by letters, as I, II, III, IV, V, . . . IX, X, etc. So named because the representation commonly used in Europe and America was borrowed from the Romans. (See page 38.)

round numbers: an approximate answer, usually expressed with zeros in the last two or three places. To express 4212 in round numbers, we should say 4200 and probably read it "forty-two hundred" instead of "four thousand two hundred".

short division: use of a divisor so small (usually under 10) that the process of division may be performed mentally. (See page 18.)

sign: See *symbol*.

subtraction: the process of taking one number away from another. (See page 9.)

subtrahend: a number which is to be subtracted from another number.

sum: a number obtained by adding together two or more numbers; sometimes used as a synonym for *answer* or *problem*.

symbol: a letter or mark representing a number, a quantity, or an operation.

table: a listing of results previously worked out or of relationships to which frequent reference is desirable, arranged in systematic form.

times: the process of multiplying; the name for the sign of multiplication.

trial divisor: an approximate number used to shorten the operation of division in a preliminary test before the whole operation is performed. (See page 20.)

unit: the name for the column in which numbers under 10 are placed in the Arabic system. (See page 39.)

unity: the whole; expressed by the number, 1, and used as synonym for 1.

value: the amount represented by a number.

vertical: extending down the page from top to bottom.